University of Antwerp
Operations Research Group
ANT/OR

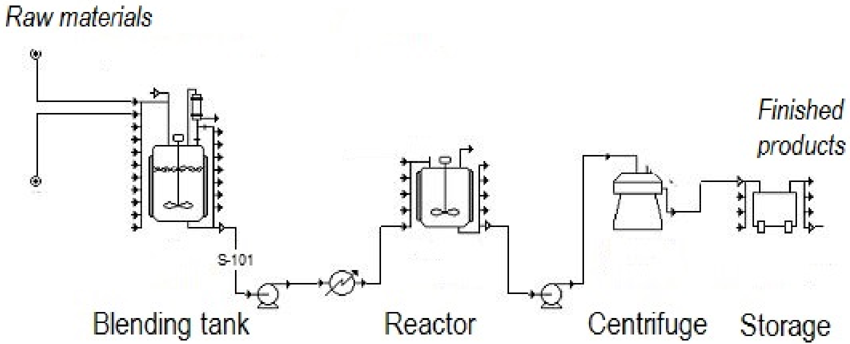
Optimal design of multiproduct batch plants using heuristic techniques

Floor Verbiest, Trijntje Cornelissens and Johan Springael
ANT/OR research group, Dept of Engineering management, University of Antwerp
Orbel31 - February 2, 2017

Outline

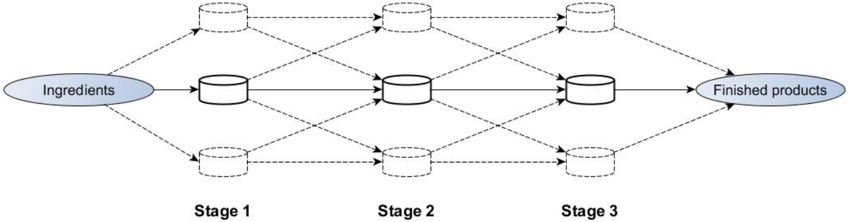
Batch process

Batch plant: Finite quantities made in discrete batches



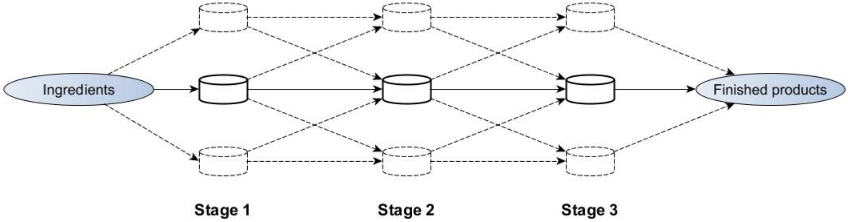
Given the products needed, recipes and production horizon:

Determine optimal **number** and **size** of batch equipment units, out of a discrete set of sizes, so as to minimise capital costs while satisfying design and horizon constraints



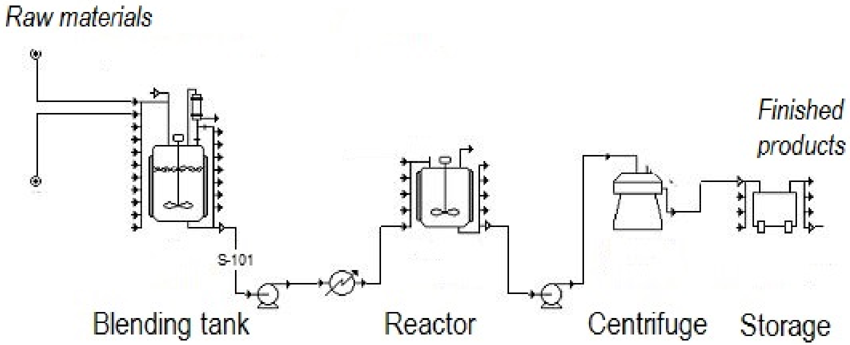
Given the products needed, recipes and production horizon:

Determine optimal number and size of batch equipment units, out of a discrete set of sizes, so as to **minimise capital costs** while satisfying **design and horizon constraints**



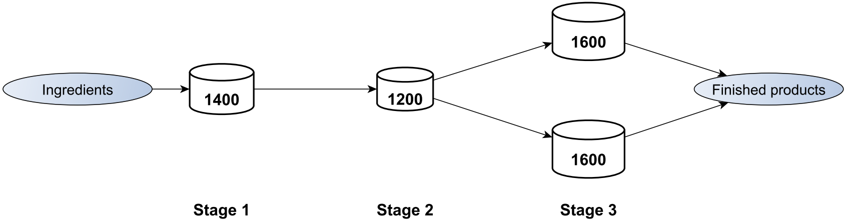
Batch process

Batch plant: Finite quantities made in discrete batches



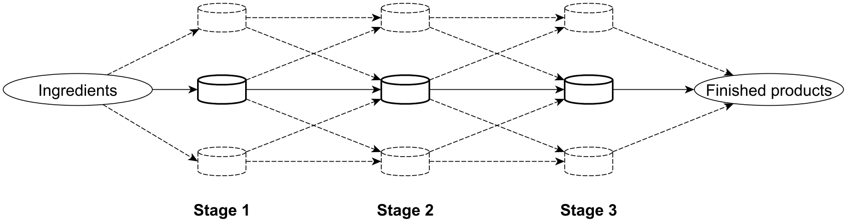
Given the products needed, recipes and production horizon:

Determine optimal **number** and **size** of batch equipment units, out of a discrete set of sizes, so as to **minimise capital costs** while satisfying **design and horizon constraints**

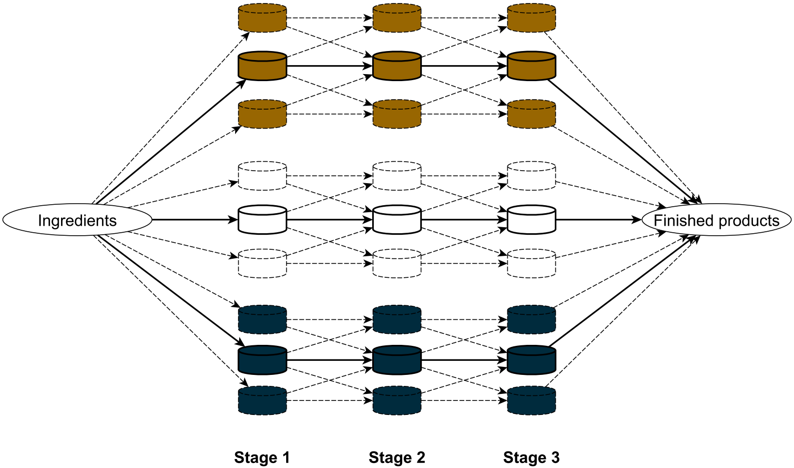


Literature - Design options
Single line

Majority of literature: MI(N)LP model - solved exactly [1, 2]



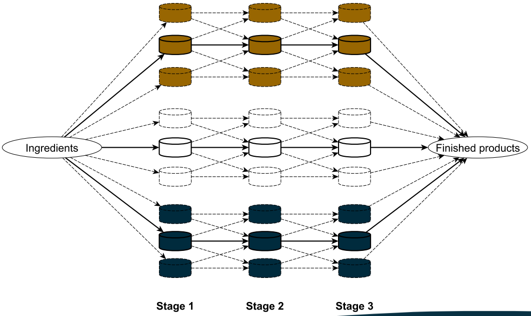
Previous study - Design options
Parallel lines [3]



Parallel production lines

Characteristics:

- Operate independently but simultaneously
- Consist of all stages
- Division of products over lines

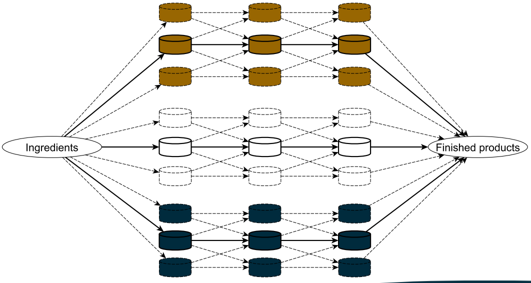


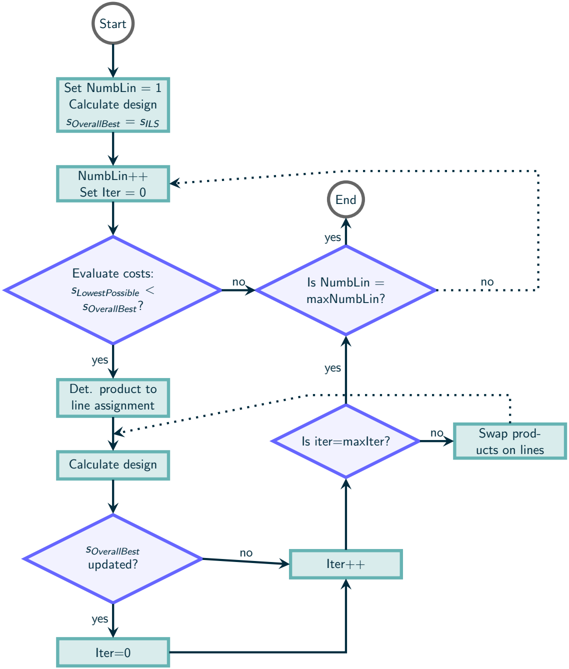
Parallel production lines

Extension of the design problem - Determine:

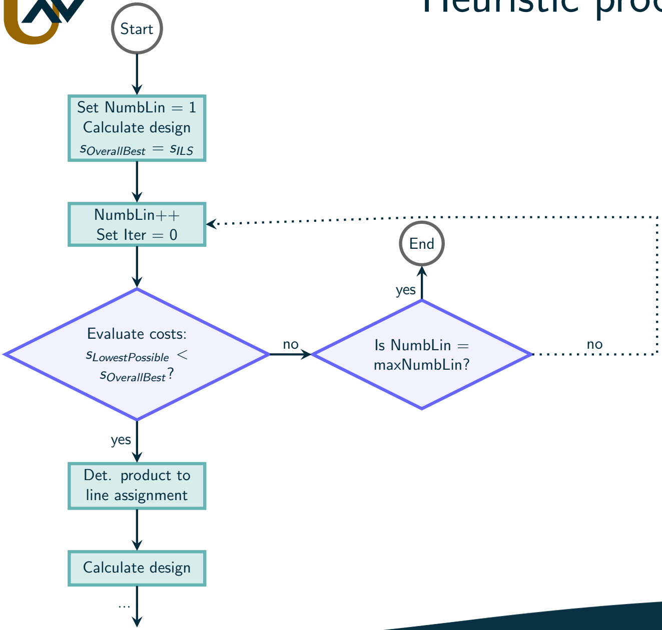
- Number of lines to install
- Number and size of equipment units per stage per installed line
- Assignment of products to installed lines

→ becomes intractable for exact solution methods when problems are larger




Start
Set NumbLin = 1
Calculate design
$s_{OverallBest} = s_{ILS}$
NumbLin++
Set Iter = 0
End
yes
Evaluate costs:
$s_{LowestPossible} <$
$s_{OverallBest}$?
no
Is NumbLin =
maxNumbLin?
no
yes
yes
Det. product to
line assignment
Is iter=maxIter?
no
Swap prod-
ucts on lines
Calculate design
$s_{OverallBest}$
updated?
no
Iter++
yes
Iter=0

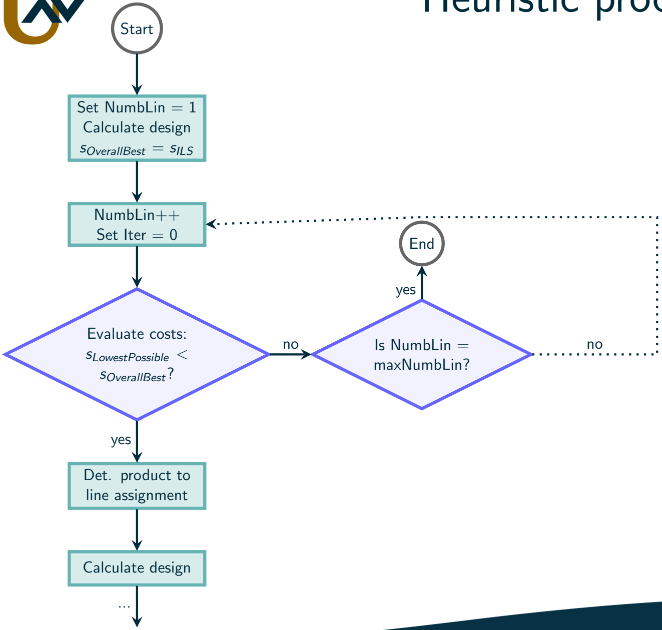
Heuristic procedure (1)



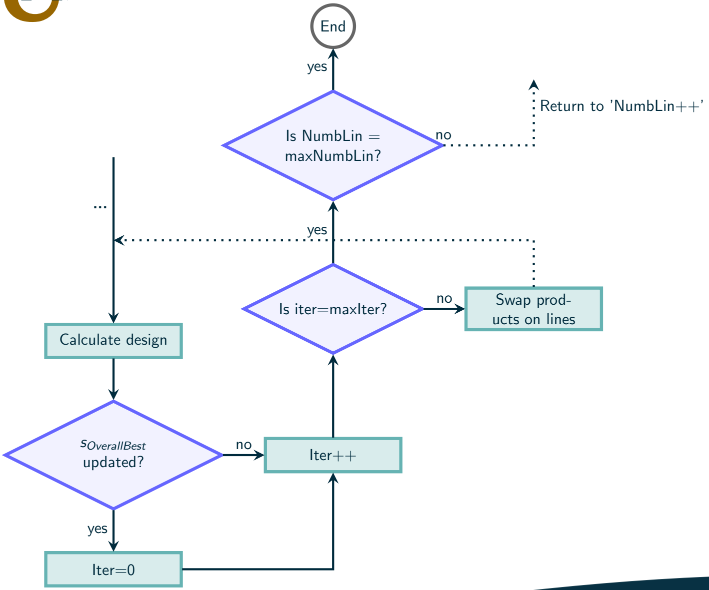
The minimisation of:

- Capital costs: cost for acquiring/installing equipment
- Startup costs: costs for preparing the equipment for the production of every product
- Contamination costs: penalisation for producing products of different families on the same equipment

Heuristic procedure (1)



Heuristic procedure (2)

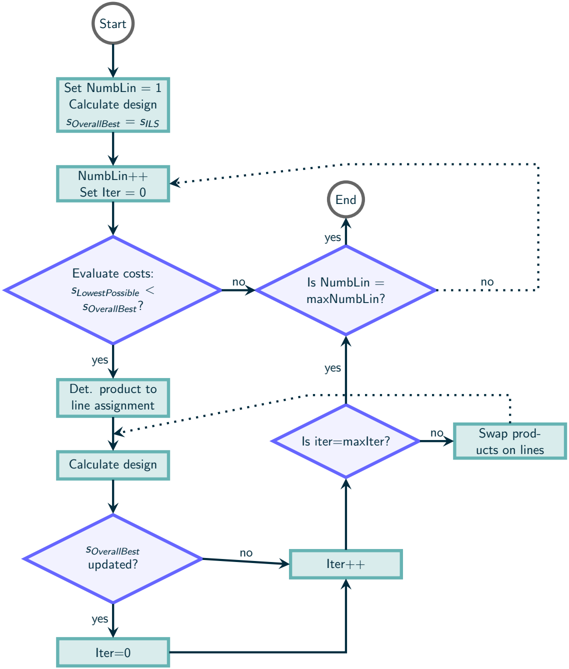


Algorithm : Iterated Local Search

$s_0 \leftarrow$ *InitialSolution*
$s \leftarrow$ *LocalSearch*(s_0)
while Stopping criterion is not met **do**
  $s' \leftarrow$ *perturbation(s)*
  s" ← LocalSearch(s').
  s ← AcceptanceCriterion(s", s).
end while

- Initial Solution: max. design (max. number of units, max. sizes)
- Local Search - moves:
  - Decrease number of units for one stage
  - Decrease size of units for one stage

  Move strategy: steepest descent
- Perturbation: reset certain % of stages to the maximum
- Termination criterion: stop after n iterations without improvement of the best solution found so far


Start
Set NumbLin = 1
Calculate design
$s_{OverallBest} = s_{ILS}$
NumbLin++
Set Iter = 0
End
yes
Evaluate costs:
$s_{LowestPossible} <$
$s_{OverallBest}$?
no
Is NumbLin =
maxNumbLin?
no
yes
yes
Det. product to
line assignment
Is iter=maxIter?
no
Swap prod-
ucts on lines
Calculate design
$s_{OverallBest}$
updated?
no
Iter++
yes
Iter=0

Results

Example 1:

- 40 products with given demand due at the end of a given horizon
- Products are divided into 4 product families
- Production process consists of 5 stages
- Possibility to install at most 6 lines
- Possibility to have max. 4 identical batch equipment units in parallel per stage
- Discrete set of sizes to choose from
- Cost coefficients are given

Results

Example 1:

- 40 products with given demand due at the end of a given horizon
- Products are divided into 4 product families
- Production process consists of 5 stages
- Possibility to install at most 6 lines
- Possibility to have max. 4 identical batch equipment units in parallel per stage

	Optimal solution (Gurobi)	Heuristic solution[1]
Example 1:		
Total costs	911 228	916 899
Time (s)	6010.0	175.9

[1] avg of 5 runs

Results

- Example 2: 20 prod., 3 prod. fam, 4 stages, max par. 5, max lin. 5
- Example 3: 50 prod., 6 prod. fam, 4 stages, max par. 6, max lin. 3
- Example 4: 150 prod., 4 prod. fam, 3 stages, max par. 8, max lin. 4

	Optimal solution (Gurobi)	Heuristic solution[2]
Example 2:		
Total costs	109 262	109 262
Time (s)	95.0	0.734
Example 3:		
Total costs	826 496	850 545
Time (s)	1275.8	424.8
Example 4:		
Total costs	- (UB: 1 062 994 - LB: 511 291)	1 018 650
Time (s)	$>$ 10h	1767.5

[2] avg of 5 runs

Conclusion

Aim: Determine design of multiproduct chemical batch plants with parallel lines

Contributions: Heuristic solution procedure

Future research:

- 'Smart' swapping
- Parameter tuning

Bibliography

[1] V. T. Voudouris and I. E. Grossmann, "Mixed-integer linear programming reformulations for batch process design with discrete equipment sizes," *Industrial & Engineering Chemistry Research*, vol. 31, no. 5, pp. 1315–1325, 1992.

[2] A. P. Barbosa-Póvoa, "A critical review on the design and retrofit of batch plants," *Computers & Chemical Engineering*, vol. 31, no. 7, pp. 833 – 855, 2007.

[3] F. Verbiest, T. Cornelissens, and J. Springael, "Design of a chemical batch plant with parallel production lines: Plant configuration and cost effectiveness," *Computers & Chemical Engineering*, vol. 99, pp. 21 – 30, 2017.

[4] H. R. Lourenço, O. C. Martin, and T. Stützle, "Iterated local search: Framework and applications," in *Handbook of Metaheuristics* (M. Gendreau and J.-Y. Potvin, eds.), vol. 146 of *International Series in Operations Research & Management Science*, pp. 363–397, Springer US, 2010.

University of Antwerp
Operations Research Group
ANT/OR

Optimal design of multiproduct batch plants using heuristic techniques

Floor Verbiest, Trijntje Cornelissens and Johan Springael
ANT/OR research group, Dept of Engineering management, University of Antwerp
Orbel31 - February 2, 2017

floor.verbiest@uantwerpen.be

University of Antwerp
Operations Research Group
ANT/OR

Appendix

Mathematical model - Single line
Basic design model - non-linear

$$\min \sum_{j=1}^{J} n_j \alpha_j V_j^{\beta_j} \qquad \alpha_j, \beta_j\text{: cost coefficients}$$

s.t.
Design constraint:

$$V_j \geqslant S_{ij} B_i \qquad \forall i, j \tag{1}$$

Horizon constraint:

$$T_i = \max_{j=1,\ldots,J} \left(\frac{\tau ij}{n_j}\right) \qquad \forall i \tag{2}$$

$$\theta_i = n_i T_i \qquad \forall i \tag{3}$$

$$\sum_{i=1}^{P} n_i T_i \leqslant H \tag{4}$$

$$Q_i = n_i B_i \qquad \forall i \tag{5}$$

Rewrite (3), (4) and (5)

$$\sum_{i=1}^{P} \frac{Q_i T_i}{B_i} \leqslant H \tag{6}$$

$V_j^{LL} \leqslant V_j \leqslant V_j^{UL} \quad \forall j;\ B_i,\ T_i \geqslant 0 \quad \forall i;\ n_j \in \mathbb{N}_0$

Mathematical model - Parallel lines
Capital costs

$$\min \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{n=1}^{N} n\alpha_j v_s^{\beta_j} y_{jsn} \qquad \alpha_j, \beta_j\text{: cost coefficients and } \beta_j < 1$$

with v_s: volume of the equipment and

$$y_{jsn} = \begin{cases} 1 & \text{if stage } j \text{ has } n \text{ units in parallel of size } s \\ 0 & \text{otherwise} \end{cases}$$

Mathematical model - Parallel lines
Startup costs

$$\min \sum_{i=1}^{P} \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{n=1}^{N} Cstart_i \, z_{ljn} t_{li}$$

with

$$z_{ljn} = \begin{cases} 1 & \text{if unit } n \text{ of stage } j \text{ of line } l \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

$$t_{li} = \begin{cases} 1 & \text{if product } i \text{ is produced on line } l \\ 0 & \text{otherwise} \end{cases}$$

and $Cstart_i$ is a line and stage independent startup cost.

Mathematical model - Parallel lines

Contamination cost

$$\min \sum_{l=1}^{L} \sum_{f=1}^{F} \sum_{j=1}^{J} \sum_{n=1}^{N} Ccont\, b_{fl} z_{ljn} g_l$$

$$b_{fl} \geqslant d_{if} t_{li} \qquad \forall l, f, i \qquad (1)$$

$$\sum_{f=1}^{F} b_{fl} \geqslant 2 - M(1 - g_l) \qquad \forall l \qquad (2)$$

$$\sum_{f=1}^{F} b_{fl} - 1 \leqslant M g_l \qquad \forall l \qquad (3)$$

with

$$b_{fl} = \begin{cases} 1 & \text{if product family } f \text{ is produced on line } l \\ 0 & \text{otherwise} \end{cases}$$

and g_l a correction binary to avoid costs if there is only 1 pf and $Ccont$ a fixed cost

Mathematical model - Parallel lines

Production line design constraints

$$z_{ljn} = \begin{cases} 1 & \text{if unit } n \text{ of stage } j \text{ of line } l \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

$$t_{li} = \begin{cases} 1 & \text{if product } i \text{ is produced on line } l \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=1}^{N} z_{lj'n} \leqslant N \sum_{n=1}^{N} z_{ljn} \qquad \forall l, j, j' \tag{4}$$

$$z_{ljn} \geqslant z_{lj(n+1)} \qquad \forall l, j, n \text{ with } n < N \tag{5}$$

$$t_{li} \leqslant \sum_{n=1}^{N} z_{ljn} \qquad \forall l, j, i \tag{6}$$

$$\sum_{l=1}^{L} t_{li} \geqslant 1 \qquad \forall i \tag{7}$$

Mathematical model - Parallel lines

Equipment design constraints

$$u_{ljs} = \begin{cases} 1 & \text{if equipment of stage } j \text{ of line } l \text{ has size } s \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=1}^{N} z_{ljn} \leqslant N \sum_{s=1}^{S} u_{ljs} \qquad \forall l, j \tag{8}$$

$$\sum_{n=1}^{N} z_{ljn} \geqslant \sum_{s=1}^{S} u_{ljs} \qquad \forall l, j \tag{9}$$

$$\sum_{s=1}^{S} u_{ljs} \leqslant 1 \qquad \forall l, j \tag{10}$$

$$n_{li} \geqslant \sum_{s=1}^{S} \frac{q_{li} S_{ij}}{v_s} u_{ljs} \qquad \forall l, j, i \tag{11}$$

Mathematical model - Parallel lines

Horizon constraints

$$\sum_{n=1}^{N} z_{ljn} T_{lji} = n_{li} \tau_{ij} \qquad \forall l, j, i \tag{12}$$

$$\theta_{li} \geqslant T_{lji} \qquad \forall l, j, i \tag{13}$$

$$\sum_{i=1}^{P} \theta_{li} \leqslant H \qquad \forall l \tag{14}$$

Mathematical model - Parallel lines
Demand constraint & Boundaries

$$\sum_{l=1}^{L} q_{li} = Q_i \qquad \forall i \tag{15}$$

$$z_{ljn}, t_{li}, u_{ljs} \in \{0, 1\} \tag{16}$$

$$q_{li}, n_{li}, T_{lji}, \theta_{li} \geqslant 0 \tag{17}$$

$$q_{li} \leqslant Q_i t_{li} \qquad \forall l, i \tag{18}$$

$$n_{li} \leqslant M_i t_{li} \qquad \forall l, i \tag{19}$$

$$T_{lji} \leqslant H t_{li} \qquad \forall l, j, i \tag{20}$$

$$\theta_{li} \leqslant H t_{lJ,i} \qquad \forall l, i \tag{21}$$

$$\epsilon t_{li} \leqslant q_{li} \qquad \forall l, i \tag{22}$$

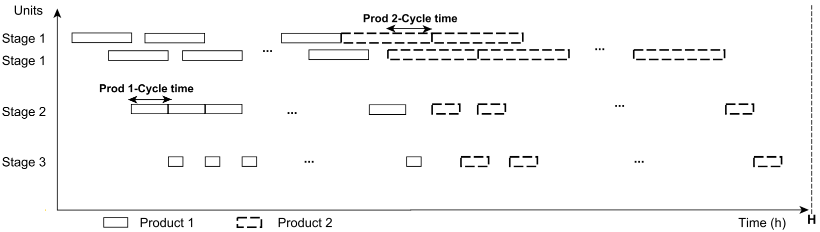
Size factor

Size factor

S_{ij}: characteristic size of equipment needed at stage j to produce unit mass of product i
$\approx$ density

$=$ since each product has different characteristics, the sizes of equipment needed to produce the same mass of product will vary from product to product

Single product campaigns



University of Antwerp
Operations Research Group
ANT/OR

Appendix

floor.verbiest@uantwerpen.be