Available online at www.sciencedirect.com
ScienceDirect
Electronic Notes in Discrete Mathematics 36 (2010) 631–638
Electronic Notes in DISCRETE MATHEMATICS
www.elsevier.com/locate/endm

# An efficient metaheuristic to improve accessibility by rural road network planning

P.A. Maya [a, b], K. Sörensen [a] and P. Goos [a, c]

[a] *Faculty of Applied Economics, University of Antwerp*

[b] *Faculty of Engineering, Universidad de Antioqua*

[c] *Erasmus School of Economics, Erasmus University Rotterdam*

**Abstract**

In this paper we consider the problem of allocating resources to upgrade a rural road network in order to improve the accessibility of as many people as possible to the main cities or regional center where the economic and social infrastructure is usually located. We propose a solution approach based on the GRASP and VNS Metaheuristic. The efficiency of our approach is demonstrated on a set of random small and medium size instances and on a large instance that has been built based on a real road network.

*Keywords:* road network planning, accessibility, GRASP.

## 1 Introduction

In rural areas of lesser-developed countries the road network plays an important role in connecting and ensuring the accessibility to the economic and social infrastructure and to facilities, such as hospitals, usually located in regional centers or in more developed cities. This role has been recognized

[1] pablo.mayaduque@ua.ac.be


doi:10.1016/j.endm.2010.05.080

by many organizations oriented towards assisting rural transport planners, rural road agencies, donor agencies, local governments, and communities in the design and appraisal of rural transport infrastructure [2,5]. *Accessibility* is defined by [2] as the degree of difficulty people or communities have in accessing locations for satisfying their basic social and economic needs. A related concept is *basic access*, which is defined by [5] as the minimum level of a rural transport infrastructure (RTI) network service required to sustain socioeconomic activity.

In this paper, we describe and solve the problem of allocating resources for upgrading some roads of a transport network, in order to allow as many people as possible to reach their closest regional center in as little time as possible. The problem we deal with can be defined on a graph that consists of a fixed set of nodes, usually population centers and also road cross points, and an underlying road network which connects those nodes. The set of nodes can be partitioned in three subsets: i) a set of regional centers, usually composed of a small group of largest or more developed cities. ii) a set of secondary cities, towns and villages and iii) a set of road cross points. The edges in the road network can also be classified in three categories: i) paved roads, ii) gravel roads and iii) trails. In fact, the number of road categories does not have to be limited to those three levels and a more extensive classification can be used if desired.

In an ideal scenario, the entire network would be upgraded so that each node in the second set $\mathcal{N}_2$ is connected with the nearest node in the first set $\mathcal{N}_1$ through paved roads. However, due to scarcity of resources, such network configuration is generally not achievable in the short time. Therefore, the objective of our problem is to look for an improvement plan that increases the accessibility of the rural village to the regional centers by upgrading gravel roads and/or trails that are not at the first level.

## 2 Problem formulation

The problem of finding an optimal improvement plan for the rural road network can be defined as follows. Let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be an undirected graph where $\mathcal{N} = \{\mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3\}$ is a node set and $\mathcal{E} = \{e_{ij} = (\nu_i, \nu_j) : \nu_i, \nu_j \in \mathcal{N}, i < j\}$ is an edge set. Nodes in $\mathcal{N}_1$ represent regional centers, nodes in $\mathcal{N}_2$ correspond to towns, and villages and nodes in $\mathcal{N}_3$ represent road cross points. Edges in $\mathcal{E}$ represent roads, each of which has a level $l_{ij}$. The level of a road is related with the condition of the road and determines the time required to traverse it. The levels are measured on a ordinal scale from 1 to $L$ where level 1 represents the best possible condition for a road. The subset $\mathcal{E}_u$ of $\mathcal{E}$ is composed of all roads that can be improved, i.e. those roads for which the initial level $l_{ij}$ is different from 1. A budget $B$ is available in order to improve the level of some roads, and an improvement cost $c_{ij}(k)$ is associated with each road $(\nu_i, \nu_j) \in \mathcal{E}_u$ and each number of levels $k$ that it can be improved compared to its current status.

For each node $i \in \mathcal{N}_2$ a measure of the accessibility is defined. That measure is the shortest travel time from $i$ to the closest regional center $j$ in $\mathcal{N}_1$, i.e. the length of the shortest path form $i$ to $j$. Additionally, a weight $w_i$ is defined for each node in $\mathcal{N}_2$ which represents the "importance" of the node. We define $w_i$ to be the number of inhabitants associated with node $i$. The objective is to minimize the weighted sum of the time to travel from each node $i$ in $\mathcal{N}_2$ to its closest regional center in $\mathcal{N}_1$. The integer decision variables $x_{ij}$ indicate the number of levels that the road between nodes $i$ and $j$ is upgraded and $SP_i(j|\mathbf{x})$ is the travel time on the shortest path between $i$ and $j$ given a solution vector $\mathbf{x}$. A mathematical integer program for the problem considered is defined as follow:

$$\min \sum_{i \in \mathcal{N}_2} \left( w_i \min_{j \in \mathcal{N}_1} \{ SP_i\,(j|\mathbf{x}) \} \right) \tag{1}$$

$$\sum_{(\nu_i,\nu_j) \in \mathcal{E}_u} c_{ij}(x_{ij}) \leq B \tag{2}$$

$$x_{ij} \leq l_{ij} - 1 \qquad \forall (\nu_i, \nu_j) \in \mathcal{E}_u \tag{3}$$

$$x_{ij} \in \mathcal{Z}^+ \qquad \forall (\nu_i, \nu_j) \in \mathcal{E}_u \tag{4}$$

## 3 A metaheuristic for the accessibility problem

We propose a metaheuristic to solve the problem of deciding which roads to upgrade and by how many levels each of the roads needs to be upgraded in such a way that the accessibility of the resulting network is maximized. Our approach is essentially a GRASP ([6]) and consists of an iteration of two phases: a greedy randomized construction phase, followed by a neighbourhood-search improvement phase.

A full description of a feasible solution for this problem, that allows us to efficiently calculate the objective function, does not only store the plan of road improvement but also the length of the shortest path from each node $i$ in $\mathcal{N}_2$ to its nearest regional center. To efficiently update the solution after a change requires us to also store the sequence of each of these shortest paths. We therefore use two square matrices to represent a feasible solution: the shortest path time ($\mathbf{T}$) and the shortest path sequence ($\mathbf{P}$). Element $T[i,j]$ represents the shortest travel time from $i$ to $j$. Element $P[i,j]$ keeps the first node of the shortest path from $i$ to $j$, which is sufficient to trace the complete path recursively.

### *3.1 Construction phase: Initial solution heuristic*

The heuristic to build an initial feasible solution is based on an insertion algorithm. This algorithm starts with an empty solution $\mathbf{x}$ where each road in $\mathcal{E}_u$ is preserved in its initial status and the remaining budget $\widehat{B}$ is equal to the total budget $B$. At the start of the insertion algorithm, matrices $\mathbf{T}$ and $\mathbf{P}$ are computed using the Floyd-Warshall algorithm described in [4]. The insertion algorithm then iteratively upgrades the level of some roads in order to improve the total accessibility and continues until no more improvements can be made within the remaining budget.

Shortest path recalculation can be required for estimating the insertion saving and for updating $\mathbf{T}$ and $\mathbf{P}$. However, in order to keep the computational effort to a minimum, these tasks are performed in an efficient way. After upgrading $e_{ij}$ it is necessary to update $\mathbf{T}$ and $\mathbf{P}$, and a procedure to recalculate the shortest paths has been implemented. That procedure evaluates for all pairs of nodes whether the path connecting them and traversing $e_{ij}$, at its new status, is shorter than the current shortest path.

The insertion algorithm described in this section is completely greedy and does not use any randomness. It therefore generates the same initial solution each time it is restarted. To be usable in our GRASP metaheuristic however, it must be able to provide various initial solutions. We therefore modify the insertion algorithm so that it selects the road to improve randomly from a restricted candidate list (RCL).

### *3.2 Improving phase: VNS*

The improving phase is based on the *variable neighborhood search* metaheuristic. It corresponds to the *basic* VNS variant described in [3]. Our VNS algorithm uses three move types, defining three different neighborhood structures. First, downgrade changes the status of a road from the current value to the next lower value. Second, the upgrade move improves the status of a road from the current value to the next larger value. Incidentally, this is the move that is used by the insertion algorithm to build the initial solution. The third move, swap, simultaneously applies the two previous moves. The neighborhood structures are used in the order suggested by [7], i.e. (1) downgrade, (2) swap and (3) upgrade.

The neighborhood defined by the downgrade move never contains a solution with a better objective function value than the current solution. In the best case, downgrading a road does not cause an increase in any of the shortest paths in the objective function. However, neighbors with the same objective function value that use less resources (i.e. less of the available budget) could be found. For exploring this neighborhood, we determine for each road $e_{ij} \in \mathcal{E}_u$ whether or not it is part of any of the shortest paths $SP_k(O_k)$ that connect each node $k \in \mathcal{N}_2$ to its nearest regional center $O_k \in \mathcal{N}_1$. If this is not the case, road $e_{ij}$ is downgraded to its original status. To update matrices $\mathbf{T}$ and $\mathbf{P}$, after downgrading edge $e_{ij}$, the *small label first* (SLF) algorithm for the shortest path described problem by [1] is used.

The second neighborhood is defined by the upgrade move and the search on it is performed as was described in Section 3.1 where it was described how an initial solution was constructed.

The third neighborhood (swap) considers all neighboring solutions that can be reached from the current solution by simultaneously upgrading one road and downgrading another, while not violating any constraints. The saving generated with this move is computed by summing the effect of the move over the shortest path from each $i \in \mathcal{N}_2$ to its nearest regional center. As computing the swap saving can require new shortest path calculations, we have implemented a procedure to estimate that saving rather than computing it exactly. The matrices $\mathbf{T}$ and $\mathbf{P}$ are updated in two steps. First, the updating procedure is applied considering only the road that is upgraded. Then, the SLF algorithm used for the *downgrade* move is applied considering only the road that is downgraded.

## 4 Experimental results

We have carried out a computational study to evaluate the performance of our metaheuristic. First, we consider several small instances to compare our approach to an exact solution method. Next, we consider a larger instance partially based on the information available for a real road network.

We have generated a set of instances of varying sizes. Our metaheuristic is compared to an exact efficient enumeration technique that we have implemented specifically for the purpose of comparison. The results on small and medium-sized instances are summarized in Table 1

Our metaheuristic provides the optimal solution for 84 out of 90 instances.

Table 1
Numerical results on small and medium-sized instances

| | |
|---|---|
| Number of experiments | 90 |
| Optimal Solutions (%) | 93.3 |
| Difference from optimum (%) | |
| Maximum | 1.61 |
| Average | 0.83 |
| Computational effort (s) | |
| Maximum | 13.81 |
| Average | 8.42 |
| Iterations to best solution | |
| Maximum | 907 |
| Average | 131.53 |

The maximum gap is 1.61%. The computing times for the metaheuristic are reasonable, even considering that 1000 iterations were performed for all instances and that the optimal solution was usually found long before the end of these iterations.

We have generated a larger instance to test our approach under real conditions. In order to approach a real-life case better, we built such instance based on the information gathered from diverse open sources for a real road network. The network was defined in a Geographical Information System using data obtained from GISDataDepot (`http://data.geocomm.com`), Mapaction (`www.mapaction.org`) and Falling Rain Genomics (`www.fallingrain.com`). Because it was not possible to obtain information concerning the cost of repairing each road, that cost was estimated as a linear function of the length. The instance we built considers 214 nodes (103 of them representing cities or towns) and 279 roads (190 with a improvable status).

We selected three cities as the regional centers. Five different scenarios were defined for the percentage of the total budget required to improve all roads that is available to execute the improvement plan. A summary of the results for those scenarios is presented in Table 2. Rows in *SP reduction* present the average percentage of the reduction in the shortest path. For each town we computed the difference between the shortest path to the nearest regional center over the current network and over the ideal network. Rows in *Gap to ideal scenario* indicate in how much these differences have been reduced on average after improving the network. The measurements we have just described are computed for the complete set of towns that can benefit from the improvement plan (*All*), the 20 most populated towns (*Popul.* $Q_4$) and the 20 remotest cities over the current network (*Dist.* $Q_4$).

Table 2
Numerical results for the real-life inspired instance

| | | | | | |
|---|---|---|---|---|---|
| Number of towns considered | | | | 100 | |
| Number of towns that can benefit | | | | 79 | |
| Budget (%) | 10 | 25 | 40 | 50 | 60 |
| Towns benefited (%) | 79.7 | 93.7 | 97.5 | 100.0 | 100.0 |
| Population benefited (%) | 83.1 | 95.0 | 97.2 | 100.0 | 100.0 |
| SP reduction (%) | | | | | |
| All towns | 24.2 | 30.6 | 34.3 | 36.0 | 36.0 |
| Popul. $Q_4$ | 30.9 | 35.9 | 37.4 | 37.8 | 37.8 |
| Dist. $Q_4$ | 24.5 | 30.5 | 33.9 | 35.5 | 35.6 |
| Gap to ideal scenario (%) | | | | | |
| All towns | 59.8 | 79.2 | 93.0 | 99.9 | 100.0 |
| Popul. $Q_4$ | 73.7 | 88.9 | 94.5 | 100.0 | 100.0 |
| Dist. $Q_4$ | 69.2 | 85.5 | 95.2 | 99.8 | 100.0 |

For the instance considered, the ideal accessibility conditions are reached with 60% of the total budget required to improve all roads. This shows that a good planning road is important to maximize the benefit when allocating scarce resources to upgrade the network. Besides, results show that our algorithm prioritizes both more populated and more remote towns in the instance considered.

# 5 Conclusions and future research

We have studied the problem of looking for an improvement plan for a road network in order to increase the accessibility of the rural villages to the regional centers by allocating resources to upgrade roads that are not at their best level. We have described and implemented a solution approach based on the GRASP and VNS metaheuristics. The approach we proposed was tested using a set of small instances and a large real life based case. An excellent performance was observed in both cases. Experiments also pointed out the importance of using OR techniques to help the decision process in network planning, as they allow to maximize the benefit for the communities by allocating optimally scarce resources. Two areas for future research arise from this work. First, one avenue for future work is to enhance the scope of the problem by considering multiple objectives and multiple periods such that resources availability and upgrading decisions can be spread over a time horizon. Second, exploring re-optimization techniques for the shortest path problem could lead to a higher efficiency of our algorithm in larger instances.

# References

[1] Bertsekas, D., "Network optimization. Continuous and discrete models", Athena Scientific, Belmont (1998), 51-114.

[2] Donnges, C., "Improving Access in Rural Areas: Guidelines for Integrated Rural Accessibility Planning ", International Labour Organization, Bangkok (2003), 116pp.

[3] Hansen, P., and N. Mladenovic, "Variable Neighborhood Search", Search Methodologies, Springer, New York (2005), 211–238.

[4] Hu, T.C., and M.T. Shing, "Combinatorial algorithms", Dover Publications, New York (2002), 10–16.

[5] Lebo, J., and D Schelling, "Design and Appraisal of Rural Transport Infrastructure Ensuring Basic Access for Rural Communities", World Bank, Washington D.C. (2001), 110pp.

[6] Resende, M., and C.C. Ribeiro, "Greedy randomized adaptive search procedures", Handbook of Metaheuristics, Kluwer Academic Publishers, (2003), 211–238.

[7] Scaparra, M., and R. Church, *A GRASP and Path Relinking Heuristic for Rural Road Network Development*, Journal of Heuristics. **11** (2005), 89-108.