University of Antwerp
Operations Research Group
ANT/OR

A matheuristic solution approach for the optimal design of multiproduct batch plants with parallel production lines

Floor Verbiest, Trijntje Cornelissens and Johan Springael
ANT/OR research group, Dept of Engineering management, University of Antwerp
IFORS - 19 July, 2017

Research funded by the Research Foundation - Flanders (FWO)

Outline

Batch plant design

Introduction parallel production lines

Matheuristic solution approach

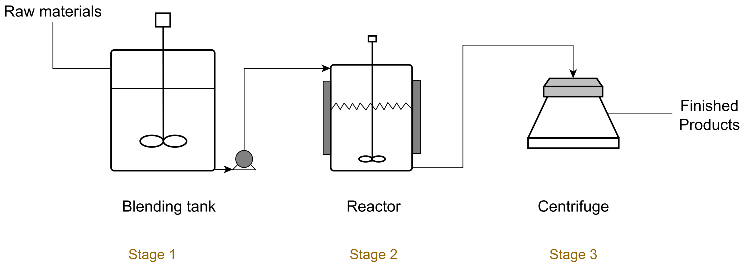
Results

Conclusion and future work

Batch process

What Production of fine and specialty chemicals:
e.g. pharmaceuticals, food additives, lubricants.

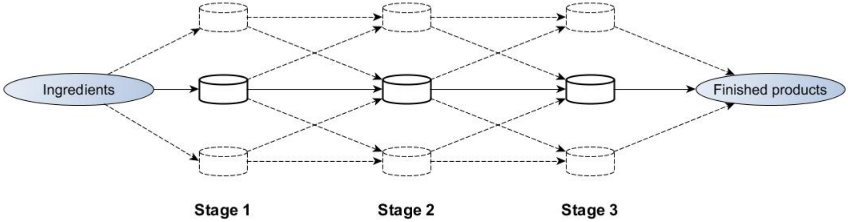
How Finite quantities made in discrete batches



Batch plant design
Optimisation problem - min. capital cost

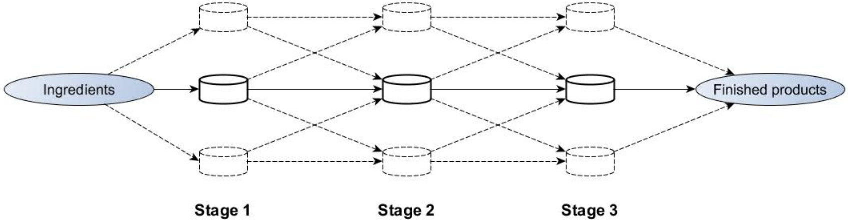
Given the product volumes needed, recipes and production horizon:

Determine optimal **number** and **size** of batch equipment units for every stage, out of a discrete set of sizes, so as to minimise capital costs while satisfying design and horizon constraints



Given the product volumes needed, recipes and production horizon:

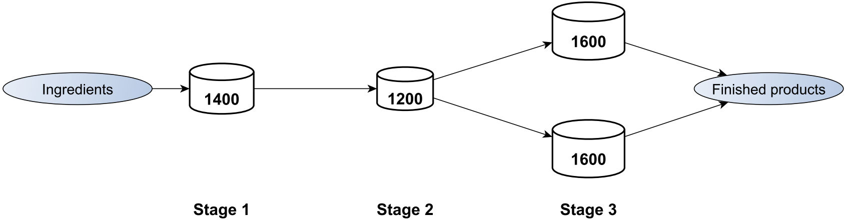
Determine optimal number and size of batch equipment units for every stage, out of a discrete set of sizes, so as to **minimise capital costs** while satisfying **design and horizon constraints**



Batch plant design
Optimisation problem - min. capital cost

Given the product volumes needed, recipes and production horizon:

Determine optimal number and size of batch equipment units for every stage, out of a discrete set of sizes, so as to minimise capital costs while satisfying design and horizon constraints

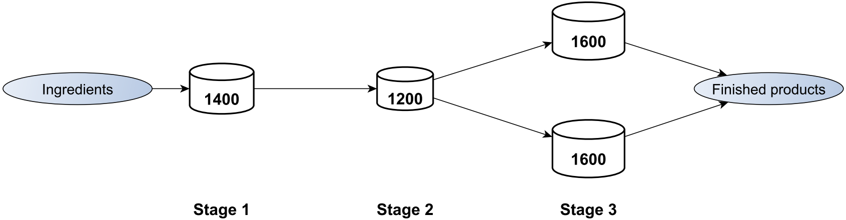


Majority of literature: MI(N)LP model - solved exactly [1, 2]

Batch plant design
Optimisation problem - min. capital cost

Given the product volumes needed, recipes and production horizon:

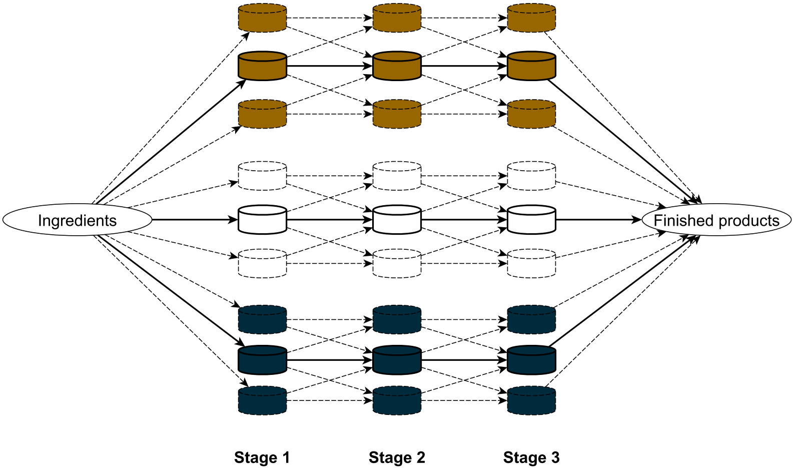
Determine optimal number and size of batch equipment units for every stage, out of a discrete set of sizes, so as to minimise capital costs while satisfying design and horizon constraints



Majority of literature: MI(N)LP model - solved exactly [1, 2]

New design option - parallel lines

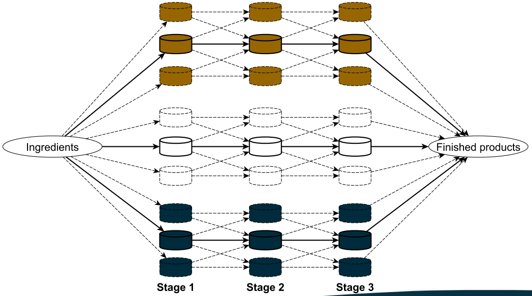
Optimisation problem - min. capital and setup costs [3]



Parallel production lines

Characteristics:

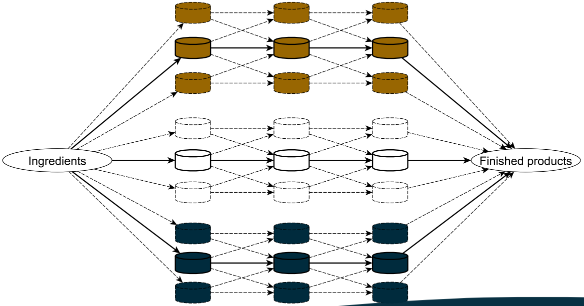
- Installed on 1 production site
- Operate independently but simultaneously
- Consist of all stages
- Distribution of products over lines



Parallel production lines

Extension of the design problem - Determine:

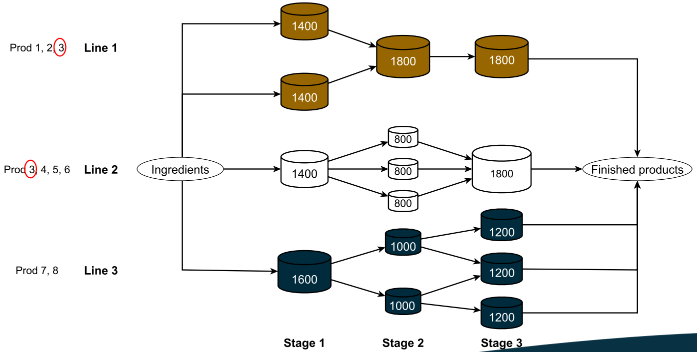
- Number of lines to install
- Number and size of equipment units per stage per installed line
- Assignment of production to installed lines
  - a product can be produced on multiple production lines (= product duplication)



Parallel production lines

Extension of the design problem - Determine:

- Number of lines to install
- Number and size of equipment units per stage per installed line
- Assignment of production to installed lines
  - a product can be produced on multiple production lines



Parallel production lines

Extension of the design problem - Determine:

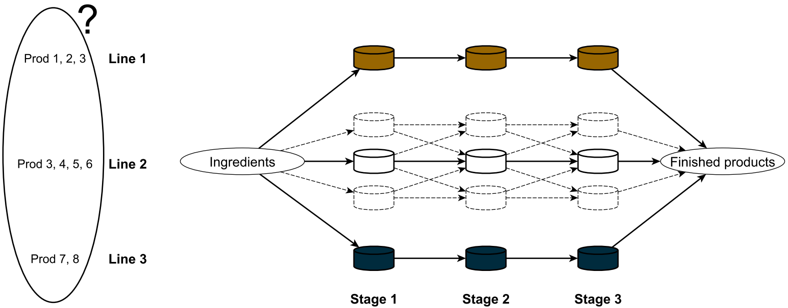
- Number of lines to install
- Number and size of equipment units per stage per installed line
- Assignment of production to installed lines
  - a product can be produced on multiple production lines

$\rightarrow$ becomes intractable for exact solution methods
when problems are larger

Decomposition heuristic: divide problem into subproblems

- Product to line assignment
- Design problem

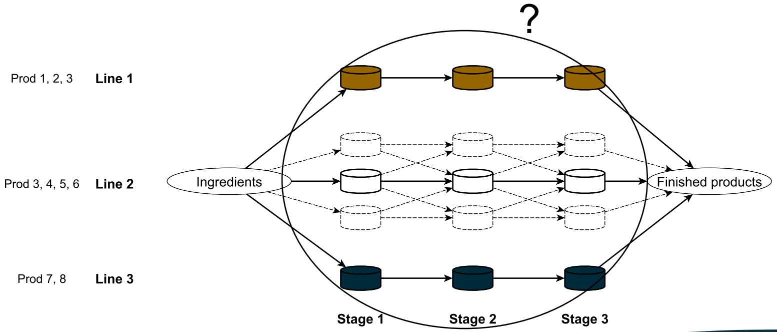
Including feedback loops



Decomposition heuristic: divide problem into subproblems

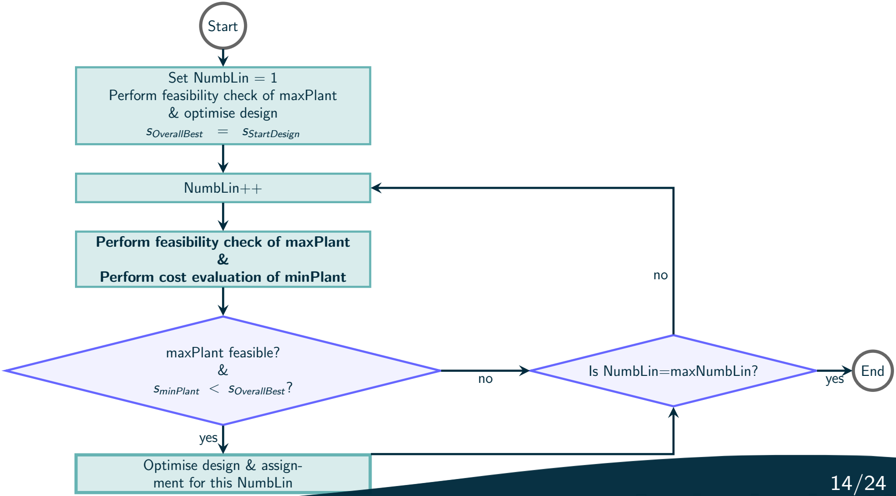
- Product to line assignment
- Design problem

Including feedback loops



Global procedure
Set number of lines

1. Single line - only design problem
2. Multiple lines - design and assignment problem
   - 2 Evaluations: feasibility and cost



Evaluate costs

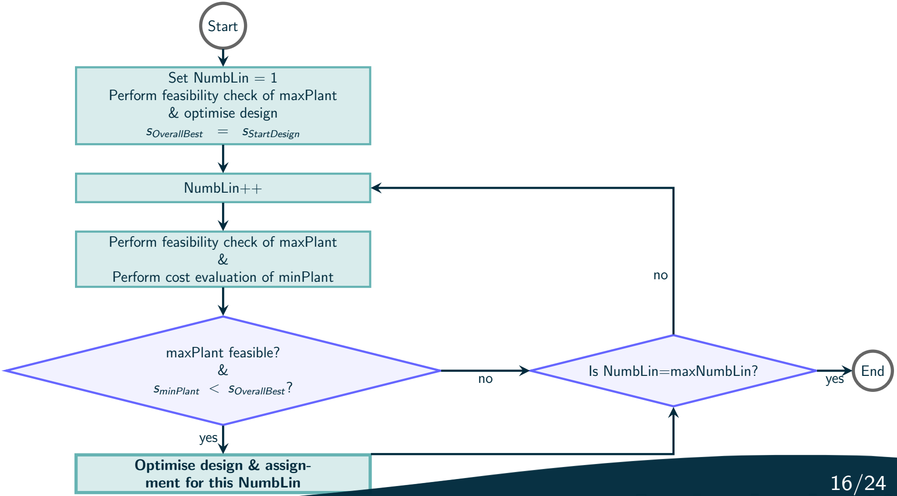
Cost evaluation of minPlant considers:

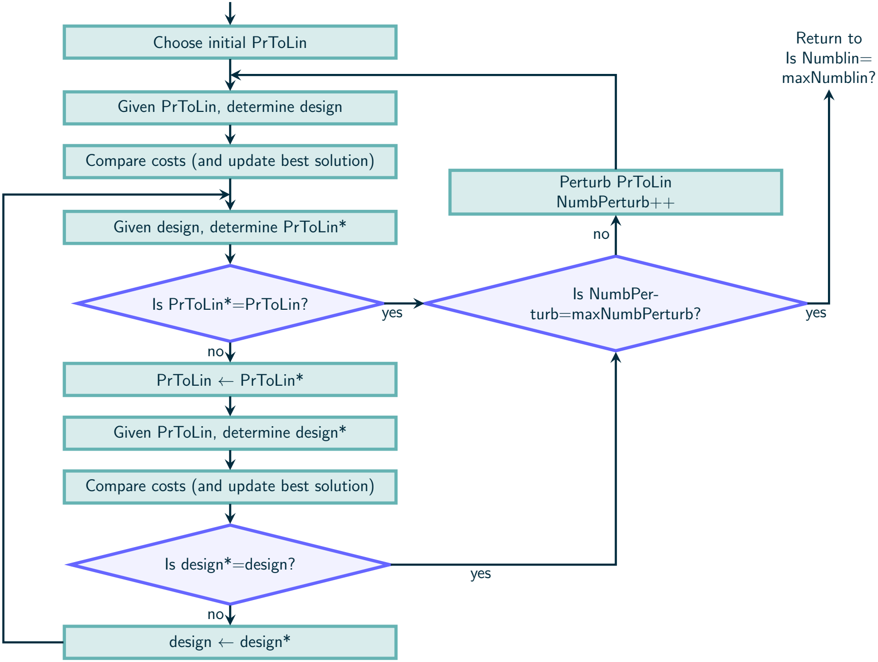
- Capital costs: cost for acquiring/installing equipment
- Startup costs: costs for preparing the equipment for the production run of every product
- Contamination costs: penalisation for producing products of different families on the same equipment

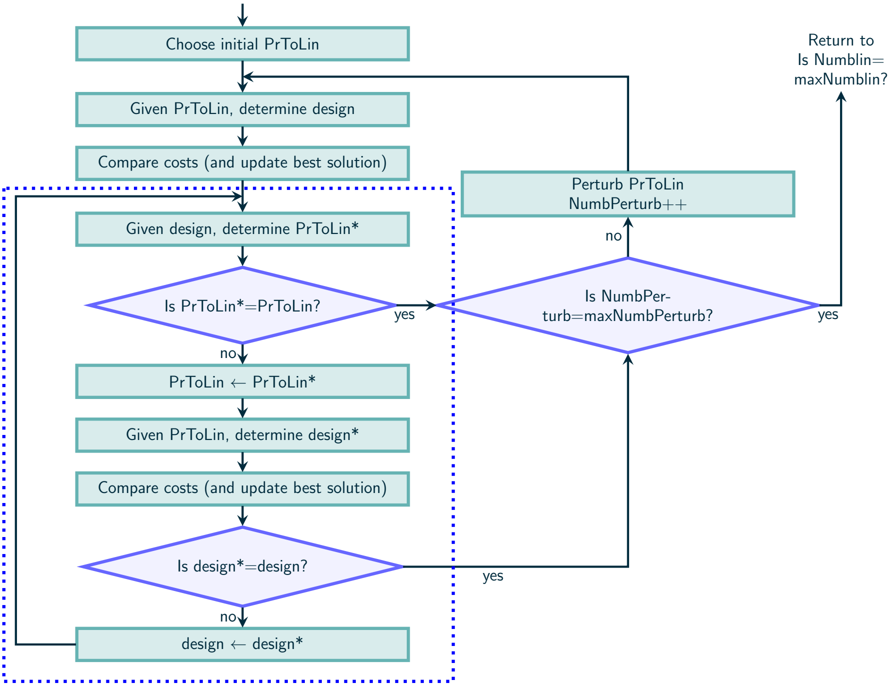
Used as objective function in later optimisations

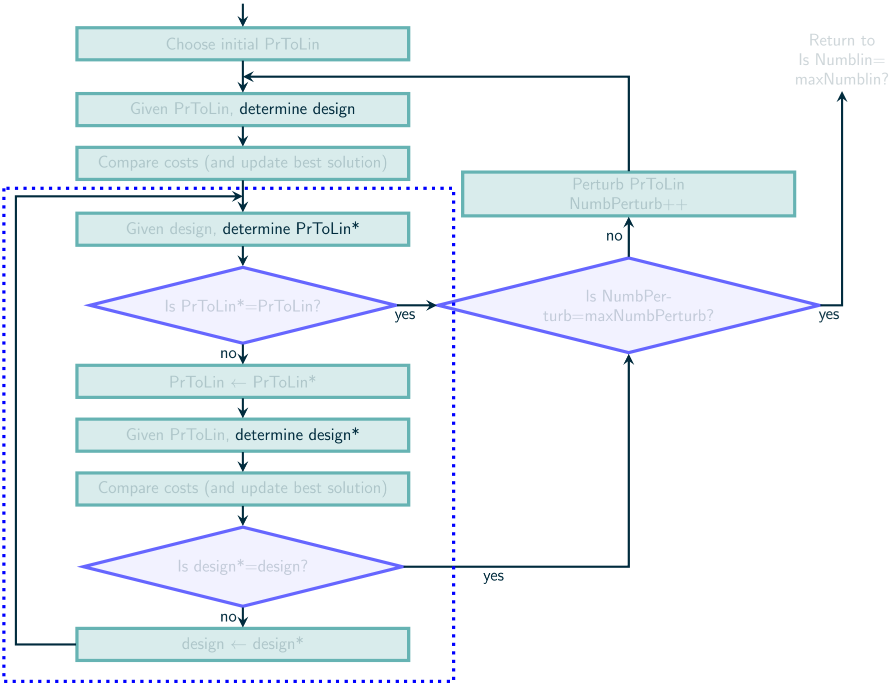
Global procedure
Set number of lines

1. Single line - only design problem
2. Multiple lines - design and assignment problem
   - 2 Evaluations: feasibility and cost




Choose initial PrToLin
Given PrToLin, determine design
Compare costs (and update best solution)
Given design, determine PrToLin*
Is PrToLin*=PrToLin?
yes
no
PrToLin ← PrToLin*
Given PrToLin, determine design*
Compare costs (and update best solution)
Is design*=design?
yes
no
design ← design*
Is NumbPer-
turb=maxNumbPerturb?
yes
no
Perturb PrToLin
NumbPerturb++
Return to
Is Numblin=
maxNumblin?

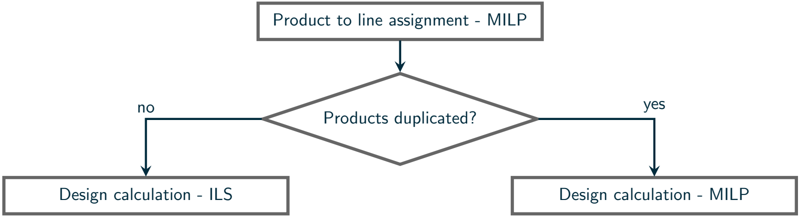

Choose initial PrToLin
Given PrToLin, determine design
Compare costs (and update best solution)
Given design, determine PrToLin*
Is PrToLin*=PrToLin?
yes
no
PrToLin ← PrToLin*
Given PrToLin, determine design*
Compare costs (and update best solution)
Is design*=design?
yes
no
design ← design*
Is NumbPerturb=maxNumbPerturb?
no
yes
Perturb PrToLin
NumbPerturb++
Return to
Is Numblin=
maxNumblin?


Choose initial PrToLin
Given PrToLin, determine design
Compare costs (and update best solution)
Given design, determine PrToLin*
Is PrToLin*=PrToLin?
yes
no
PrToLin ← PrToLin*
Given PrToLin, determine design*
Compare costs (and update best solution)
Is design*=design?
yes
no
design ← design*
Perturb PrToLin
NumbPerturb++
no
Is NumbPerturb=maxNumbPerturb?
yes
Return to
Is Numblin=
maxNumblin?

Solution techniques

Solve each subproblem:

- Product to line assignment, with given design: MILP, exact solver
- Design problem, with given line assignment
  - Metaheuristic: Iterated local search [4, 5]
  - MILP, exact solver



Results

	Opt. sol. (Gurobi)	CPU (sec)	Best math. sol.[1]	gap (%)	CPU (sec)
10 products					
Ca.	128 808	351.1	128 808	**0**	19.64
Ca.St.	166 947	5230.5	167 281	0.2	0.58
Ca.St.Co.	168 588	38.0	168 588	**0**	1.73
28 products					
Ca.	115 065	213.74	115 065	**0**	42.37
Ca.St.	173 035	661.47	173 035	**0**	1.60
Ca.St.Co.	180 747	1179.85	181 035	0.2	4.97
40 products					
Ca.	96 517	3551.62	96 517	**0**	262.14
Ca.St.	996 517	4892.11	996 517	**0**	1.85
Ca.St.Co.	1 016 517	5282.02	1 016 517	**0**	1.74
50 products					
Ca.	432 133	516.08	432 133	**0**	0.03
Ca.St.	797 696	313.15	797 696	**0**	1.91
Ca.St.Co.	826 496	912.57	826 496	**0**	2.55
60 products					
Ca.	6958	753.87	6958	**0**	0.03
Ca.St.	24 958	819.74	24 958	**0**	0.03
Ca.St.Co.	26 158	1543.72	26 165	0.02	0.04
75 products					
Ca.	1 872 676 (gap: 85%)	14 400	1 759 634	-	6058.09
Ca.St.	no feas sol (LB:358 081)	14 400	1 981 117	-	1388.46
Ca.St.Co.	no feas sol (LB: 374 894)	14 400	2 013 775	-	4509.8
150 products					
Ca.	336 689 (gap: 89 %)	14 400	228 944	-	7847.64
Ca.St.	20 932 033 (gap: 97%)	14 400	822 612	-	370.99
Ca.St.Co.	23 620 033 (gap: 98%)	14 400	1 001 958	-	65.47

[1] out of 10 runs

Conclusion

Aim: Determine design of multiproduct chemical batch plants with parallel lines

Contributions: An efficient matheuristic is developed, for solving the batch plant design problem

Future research:

- Improve MILP calculations
- Thoroughly test algorithm

Bibliography

[1] V. T. Voudouris and I. E. Grossmann, "Mixed-integer linear programming reformulations for batch process design with discrete equipment sizes," *Industrial & Engineering Chemistry Research*, vol. 31, no. 5, pp. 1315–1325, 1992.

[2] A. P. Barbosa-Póvoa, "A critical review on the design and retrofit of batch plants," *Computers & Chemical Engineering*, vol. 31, no. 7, pp. 833 – 855, 2007.

[3] F. Verbiest, T. Cornelissens, and J. Springael, "Design of a chemical batch plant with parallel production lines: Plant configuration and cost effectiveness," *Computers & Chemical Engineering*, vol. 99, pp. 21 – 30, 2017.

[4] H. R. Lourenço, O. C. Martin, and T. Stützle, "Iterated local search: Framework and applications," in *Handbook of Metaheuristics* (M. Gendreau and J.-Y. Potvin, eds.), vol. 146 of *International Series in Operations Research & Management Science*, pp. 363–397, Springer US, 2010.

[5] K. Sörensen and F. W. Glover, *Metaheuristics*, pp. 960–970. Boston, MA: Springer US, 2013.

University of Antwerp
Operations Research Group
ANT/OR

A matheuristic solution approach for the optimal design of multiproduct batch plants with parallel production lines

Floor Verbiest, Trijntje Cornelissens and Johan Springael
ANT/OR research group, Dept of Engineering management, University of Antwerp
IFORS - 19 July, 2017

Research funded by the Research Foundation - Flanders (FWO)

floor.verbiest@uantwerpen.be

University of Antwerp
Operations Research Group
ANT/OR

Appendix

Calculate Design
Iterated Local Search [1]

Algorithm : Iterated Local Search

$s_0 \leftarrow InitialSolution$
$s \leftarrow LocalSearch(s_0)$
while Termination criterion is not met **do**
    $s' \leftarrow$ *perturbation(s)*
    s" $\leftarrow$ *LocalSearch(s')*.
    s $\leftarrow$ *AcceptanceCriterion(s", s*)*.
end while

- Initial Solution: max. design (max. number of units, max. sizes)
- Local Search - moves:
  - Decrease number of units for one stage
  - Decrease size of units for one stage

  Move strategy: steepest descent
- Perturbation: reset certain % of stages to the maximum
- Termination criterion: stop after n iterations without improvement of the best solution found so far

Mathematical model - Single line
Basic design model - non-linear

$$\min \sum_{j=1}^{J} n_j \alpha_j V_j^{\beta_j} \qquad \alpha_j, \beta_j\text{: cost coefficients}$$

s.t.
Design constraint:

$$V_j \geqslant S_{ij} B_i \qquad \forall i, j \tag{1}$$

Horizon constraint:

$$T_i = \max_{j=1,\dots,J} \left(\frac{\tau ij}{n_j}\right) \qquad \forall i \tag{2}$$

$$\theta_i = n_i T_i \qquad \forall i \tag{3}$$

$$\sum_{i=1}^{P} n_i T_i \leqslant H \tag{4}$$

$$Q_i = n_i B_i \qquad \forall i \tag{5}$$

Rewrite (3), (4) and (5)

$$\sum_{i=1}^{P} \frac{Q_i T_i}{B_i} \leqslant H \tag{6}$$

$V_j^{LL} \leqslant V_j \leqslant V_j^{UL} \quad \forall j;\ B_i,\ T_i \geqslant 0 \quad \forall i;\ n_j \in \mathbb{N}_0$

Mathematical model - Parallel lines
Capital costs

$$\min \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{s=1}^{S} \sum_{n=1}^{N} \alpha_j v_s^{\beta_j} z_{ljn} u_{ljs}$$

α_j, β_j: cost coefficients and $\beta_j < 1$

with v_s: volume of the equipment and

$$z_{ljn} = \begin{cases} 1 & \text{if unit } n \text{ of stage } j \text{ of line } l \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

and

$$u_{ljs} = \begin{cases} 1 & \text{if equipment of stage } j \text{ of line } l \text{ has size } s \\ 0 & \text{otherwise} \end{cases}$$

$$\min \sum_{i=1}^{P} \sum_{l=1}^{L} \sum_{j=1}^{J} \sum_{n=1}^{N} Cstart_i \, z_{ljn} t_{li}$$

with

$$z_{ljn} = \begin{cases} 1 & \text{if unit } n \text{ of stage } j \text{ of line } l \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

$$t_{li} = \begin{cases} 1 & \text{if product } i \text{ is produced on line } l \\ 0 & \text{otherwise} \end{cases}$$

and $Cstart_i$ is a line and stage independent startup cost.

Mathematical model - Parallel lines
Contamination cost

$$\min \sum_{l=1}^{L} \sum_{f=1}^{F} \sum_{j=1}^{J} \sum_{n=1}^{N} Ccont\, b_{fl} z_{ljn} g_l$$

$$b_{fl} \geqslant d_{if} t_{li} \qquad \forall l, f, i \qquad (1)$$

$$\sum_{f=1}^{F} b_{fl} \geqslant 2 - M(1 - g_l) \qquad \forall l \qquad (2)$$

$$\sum_{f=1}^{F} b_{fl} - 1 \leqslant M g_l \qquad \forall l \qquad (3)$$

with

$$b_{fl} = \begin{cases} 1 & \text{if product family } f \text{ is produced on line } l \\ 0 & \text{otherwise} \end{cases}$$

and g_l a correction binary to avoid costs if there is only 1 pf and *Ccont* a fixed cost

Mathematical model - Parallel lines

Production line design constraints

$$z_{ljn} = \begin{cases} 1 & \text{if unit } n \text{ of stage } j \text{ of line } l \text{ exists} \\ 0 & \text{otherwise} \end{cases}$$

$$t_{li} = \begin{cases} 1 & \text{if product } i \text{ is produced on line } l \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=1}^{N} z_{lj'n} \leqslant N \sum_{n=1}^{N} z_{ljn} \qquad \forall l, j, j' \tag{4}$$

$$z_{ljn} \geqslant z_{lj(n+1)} \qquad \forall l, j, n \text{ with } n < N \tag{5}$$

$$t_{li} \leqslant \sum_{n=1}^{N} z_{ljn} \qquad \forall l, j, i \tag{6}$$

$$\sum_{l=1}^{L} t_{li} \geqslant 1 \qquad \forall i \tag{7}$$

Mathematical model - Parallel lines
Equipment design constraints

$$u_{ljs} = \begin{cases} 1 & \text{if equipment of stage } j \text{ of line } l \text{ has size } s \\ 0 & \text{otherwise} \end{cases}$$

$$\sum_{n=1}^{N} z_{ljn} \leqslant N \sum_{s=1}^{S} u_{ljs} \qquad \forall l, j \tag{8}$$

$$\sum_{n=1}^{N} z_{ljn} \geqslant \sum_{s=1}^{S} u_{ljs} \qquad \forall l, j \tag{9}$$

$$\sum_{s=1}^{S} u_{ljs} \leqslant 1 \qquad \forall l, j \tag{10}$$

$$n_{li} \geqslant \sum_{s=1}^{S} \frac{q_{li} S_{ij}}{v_s} u_{ljs} \qquad \forall l, j, i \tag{11}$$

Mathematical model - Parallel lines
Horizon constraints

$$\sum_{n=1}^{N} z_{ljn} T_{lji} = n_{li} \tau_{ij} \qquad \forall l, j, i \tag{12}$$

$$\theta_{li} \geqslant T_{lji} \qquad \forall l, j, i \tag{13}$$

$$\sum_{i=1}^{P} \theta_{li} \leqslant H \qquad \forall l \tag{14}$$

Mathematical model - Parallel lines

Demand constraint & Boundaries

$$\sum_{l=1}^{L} q_{li} = Q_i \qquad \forall i \tag{15}$$

$$z_{ljn}, t_{li}, u_{ljs} \in \{0, 1\} \tag{16}$$

$$q_{li}, n_{li}, T_{lji}, \theta_{li} \geqslant 0 \tag{17}$$

$$q_{li} \leqslant Q_i t_{li} \qquad \forall l, i \tag{18}$$

$$n_{li} \leqslant M_i t_{li} \qquad \forall l, i \tag{19}$$

$$T_{lji} \leqslant H t_{li} \qquad \forall l, j, i \tag{20}$$

$$\theta_{li} \leqslant H t_{lJ,i} \qquad \forall l, i \tag{21}$$

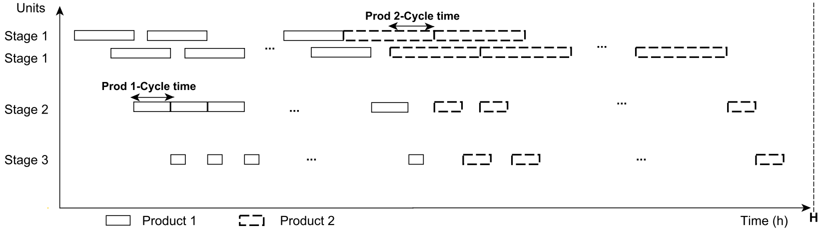
$$\epsilon t_{li} \leqslant q_{li} \qquad \forall l, i \tag{22}$$

Size factor

S_{ij}: characteristic size of equipment needed at stage j to produce unit mass of product i
$\approx$ density

$=$ since each product has different characteristics, the sizes of equipment needed to produce the same mass of product will vary from product to product

Single product campaigns



Bibliography

[1] H. R. Lourenço, O. C. Martin, and T. Stützle, "Iterated local search: Framework and applications," in *Handbook of Metaheuristics* (M. Gendreau and J.-Y. Potvin, eds.), vol. 146 of *International Series in Operations Research & Management Science*, pp. 363–397, Springer US, 2010.

University of Antwerp
Operations Research Group
ANT/OR

Appendix

floor.verbiest@uantwerpen.be