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Decision Support

Measuring and rewarding flexibility in collaborative distribution, including two-partner coalitions ☆

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ABSTRACT

Horizontal collaboration among shippers is gaining traction as a way to increase logistic efficiency. The total distribution cost of a logistic coalition is generally between 9% and 30% lower than the sum of costs of each partner distributing separately. However, the coalition gain is highly dependent on the flexibility that each partner allows in its delivery terms. Flexible delivery dates, flexible order sizes, order splitting rules, etc., allow the coalition to exploit more opportunities for optimization and create better and cheaper distribution plans.

An important challenge in a logistic coalition is the division (or sharing) of the coalition gain. Several methods have been proposed for this purpose, often stemming from the field of game theory. This paper states that an adequate gain sharing method should not only be fair, but should also reward flexibility in order to persuade companies to relax their delivery terms. Methods that limit the criteria for cost allocation to the marginal costs and the values of the subcoalitions are found to be able to generate adequate incentives for companies to adopt a flexible position. In a coalition of two partners however, we show that these methods are not able to correctly evaluate an asymmetric effort to be more flexible. For this situation, we suggest an alternative approach to better measure and reward the value of flexibility.

1. Introduction

Horizontal collaboration is defined as collaboration that occurs between companies that operate on the same level of the supply chain (European Commission, 2011). Horizontal logistic collaboration can take on many forms (Verstrepen, Cools, Cruijssen, & Dullaert, 2009). The focus in this paper is on coalitions in which several shippers outsource the delivery of their goods to a single third-party logistics provider (3PL). The 3PL organizes the delivery of the orders of all companies, and all companies allow their orders to be distributed in the same trucks as those of their partners. This strategy differs from a simple bundling of orders by the logistics service provider itself, because the benefits, costs and risks are shared among the partners, and the long-term-nature and commitment of a horizontal logistic coalition allow for continuous improvement (Slone, Dittman, & Mentzer, 2010).

One of the main positive effects of horizontal logistic collaboration is the achievement of economies of scale by transporting more volume in each trip and reducing the number of redundant trips. Several successful pilot cases have been started, that prove that this concept is viable. Examples are collaborative networks of inland waterways (Wiegmans, 2005), but also consumer goods manufacturers that optimize their distribution networks collaboratively (Bahrami, 2002), as well as 3PLs (Cruijssen, Cools, & Dullaert, 2007). Other applications vary from wood bartering in Sweden (Frisk, Göthe-Lundgren, Jörnsten, & Rönnqvist, 2010) to combining long-haul shipments from a plastic manufacturer and a steel-manufacturer from Germany to the Czech Republic (Verstrepen & 't Hooft, 2011), and horizontal collaboration among airline carriers (Oum, Park, Kim, & Yu, 2004). Moreover, there are an increasing number of papers creating the necessary frameworks for horizontal collaboration. They address issues such as the role of third party logistics providers in collaborative networks (Stefansson, 2006), the estimation of risk, benefits and environmental impact, and a multi-criteria method to support decision-making in collaborative urban freight systems (Gonzalez-Feliu & Salanova, 2012), and coordination mechanisms and benefit sharing (Audy, Lehoux, D'Amours, & Rönnqvist, 2010). However, many barriers still impede a widespread adoption of horizontal collaboration. Cruijssen, Dullaert, and Joro (2006) list the following main impediments: "finding and trusting appropriate partners", "determining and dividing the gains", "difficulties during the negotiation process", and "the absence of the right coordination and ICT-mechanisms".

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In this paper, the focus is on the second impediment listed by Cruijssen et al. (2006), i.e. determining and dividing the coalition gains. In general the total distribution cost of a coalition that bundles orders is significantly lower than the sum of the individual companies' costs. This is due to a more effective use of truck capacity, or, when using a logistics service provider, better rates due to higher volumes. A challenge is that the difference between the sum of all stand-alone costs and the coalition cost (i.e., the coalition *gain*) has to be distributed back to the partners. For this purpose, a *cost allocation method* has to be used. In Section 2 we give an overview of cost allocation methods found in the literature.

An important aspect influencing the coalition gain is the flexibility in delivery terms allowed by the partners. Allowing deliveries to be shifted in time rather than specifying a precise delivery date, allowing the pallets or boxes of a single order to be split across multiple trucks rather than forcing them to be delivered in the same truck, and so on, are good examples of such flexible delivery terms. All contribute to the optimization opportunities for the coalition and thus lead to a larger consolidation gain.

Companies that relax their delivery terms contribute more to the total reduction in cost than companies that do not. This paper states that, in order to encourage flexibility, such partners should therefore be awarded a larger portion of the gain (or, should be allocated a smaller cost). We find that methods limiting the criteria for cost allocation to the marginal costs and the values of the subcoalitions are the most adequate cost allocation methods to reward flexibility. This is demonstrated in Section 3.

In Section 4 however, we show that in small coalitions in which the effort delivered to be flexible is asymmetric and flexibility is perceived as having some (perhaps hidden) cost, those methods can easily be perceived as unfair. For this case, we develop a method to more accurately measure the added value for the coalition of a partner relaxing its delivery terms and changing from a rigid position to a flexible one. Section 5 presents some conclusions and remarks.

2. Cost allocation methods and fairness criteria

Although intuitively clear, an operational definition of the concept of *fairness* is difficult to create. Moreover, fairness may be perceived differently by different partners in a strategic coalition. Still, the literature on co-operative game theory has developed a number of characteristics (fairness criteria) that a cost allocation (or gain sharing) method should possess in order to be considered "fair".

Leng and Parlar (2005) give an overview of papers in which co-operative game theory is used in supply chain collaboration problems. After a thorough review of the literature, the authors demonstrate that collaborative supply chains present a perfect application for game theory. Collaborative supply chains consist of companies that make their own decisions, but doing so, influence the total supply chain performance. Co-operative game theory correctly assumes that collaboration will yield gains when compared to each company working individually, and focuses on how to create and divide these gains.

The concepts from game theory can readily be transferred to the setting of collaborative distribution. Given is a set of $|N|$ companies (players i), each having a stand-alone distribution cost $c(i)$, representing the cost that has to be paid by company i to deliver all its orders. The grand coalition N is defined as the coalition of all companies.

For any (sub)coalition (or group) $S \subseteq N$, there exists a distribution cost $c(S)$ that has to be paid in order to deliver all the orders of all the companies in the coalition. In this case, we assume that the distribution cost of a (sub)coalition is equal to the sum of the cost of all trips needed to deliver the pallets q of all partners of that (sub)coalition. A profit $v(S) \geqslant 0$ is defined as the difference between the sum of the stand-alone distribution costs and the global coalition distribution cost, i.e., $v(S) = \sum_{i \in S} c(i) - c(S)$. The profit of a partner working alone is thus $v(i) = 0$. This profit can be achieved by bundling orders, i.e., by allowing orders of different companies to be transported in the same trips. We assume subadditivity ($c(i+j) \leqslant c(i) + c(j), \forall i, j$), which implies that a player can not add more cost to the coalition than its original stand-alone cost.

The aim of a cost allocation method is to divide the total cost of the grand coalition $c(N)$ in such a way that each player i pays an individual cost φ_i and considers this to be fair. The difference between a company's stand-alone cost and its allocated cost ($c(i) - \varphi_i$) is called its *gain*. Because the sum of all gains is equal to the difference between the sum of all stand-alone costs and the total coalition cost, allocating the total coalition cost is equivalent to allocating the total coalition gain. For this reason, cost allocation methods are sometimes called *gain sharing* methods. In practice, cost allocation or gain sharing is done by a method (which may be a simple rule) that is agreed upon by all partners of the coalition. For an overview of the notations and characteristics of the cost and profit function, see Table 1.

The most fundamental axioms of co-operative game theory, state that a cost allocation should satisfy *Pareto-efficiency* and *individual rationality*. The first axiom enforces that the allocation is such that no player can reduce its costs without adding additional costs to the other players. The latter requires that no player will benefit from working alone and will therefore refuse to collaborate. When these axioms are fulfilled, such an allocation is generally called an *imputation*. As this is not guaranteed for all methods, we use the more general term allocation (Moulin, 1988).

Frisk et al. (2010) sum up the following list of possible cost allocation methods in horizontal logistical coalitions:

- *Activity-Based Costing* (ABC) allocates the coalition cost according to different cost drivers and activities. Investigating which activities cause costs and how to divide those costs is often time-consuming. In this paper, we assume that an activity is a trip executed and the cost driver is the number of trips, as well as the number of pallets in that trip. This means that per trip, we will allocate the costs of that specific trip proportionally to the number of pallets that a company has in that trip.

$$\varphi_i^{\mathrm{ABC}} = \sum_t \frac{q_{i,t}}{\sum_{j \in N} q_{j,t}} \tag{1}$$

- The *Equal Charge Method* allocates the *separable costs* (i.e., the marginal costs of each partner joining the final coalition) in their totality. The *non-separable costs* (the remaining costs) is allocated in an equal way.

Table 1
Important notations and the characteristics of the cost and profit function.

Notation	
N	The grand coalition
$S \subseteq N$	A subcoalition
$c(S)$	Cost of subcoalition S
$v(S)$	Profit in subcoalition S
φ_i	Allocated cost to partner i
$q_{i,t}$	Number of pallets of partner i in trip t
Characteristic function	
$\sum_{i \in S} \sum_{t \in S} q_{i,t} = c(S) \quad \forall S$	
$v(S) = \sum_{i \in S} c(i) - c(S) \quad \forall S$	
$v(S) \geqslant 0 \quad \forall S$	
$c(i+j) \leqslant c(i) + c(j) \quad \forall i, j$	

$$\varphi_i^{\mathrm{ECM}} = c(N) - c(N \setminus i) + \frac{c(N) - \sum_{j \in N}[c(N) - c(N \setminus j)]}{|N|} \tag{2}$$

- The *Volume*-method allocates the cost based on each partner's share of the total number of pallets. It differs from our calculation of ABC as it considers the *total* number of pallets and the *total* cost.

$$\varphi_i^{\mathrm{Volume}} = \frac{\sum_t q_{i,t}}{\sum_{j \in N} \sum_t q_{j,t}} c(N) \tag{3}$$

- The *Shapley value* (Shapley, 1953) takes the weighted average of a player's marginal cost in each possible subcoalition.

$$\varphi_i^{\mathrm{Shapley}} = \sum_{S \subseteq N \setminus i} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \times (c(S \cup i) - c(S)) \tag{4}$$

- The *Nucleolus* tries to find an allocation that will ensure that no partner will do better by breaking the coalition and entering a subcoalition. This property is called *stability*. Out of the set of stable profit allocations, called *the core*, the Nucleolus chooses the allocation that is found in the centre of the core. The more general characterization of Moulin (1988), which also includes games with an empty core, states that the Nucleolus shows an egalitarian concern for excesses of various coalitions. This is calculated as follows. Given any profit allocation x_i, the *excess* of company i is defined as $x_i - v(S), i \in S$, or, the difference in consolidation profit in the final coalition versus the profit of a subcoalition containing company *i*. By solving a series of linear programs (see (5)) maximizing the smallest coalitional excess, it is possible to find an allocation where for each subcoalition, this allocation is preferred by all partners and where the excesses of various coalitions are divided as equally as possible (Schmeidler, 1969). That allocation is the Nucleolus.

Given that B is the set of vectors with efficient profit allocations x, $\sum_i x_i = v(N)$

$$\max_{x \in B} \left[\min_{S \subset N} \left(\sum_{i \in S} x_i - v(S) \right) \right] \tag{5}$$

Table 2 indicates whether these cost allocation methods possess a small selection of properties found in Moulin (1988). Often, a cost allocation method requires additional information in order to be computable. ABC, for example, requires more details of the operational plan (how many pallets of each company are in how many trips ($q_{i,t}$), the cost of a trip cost ($c(t)$)). The additional information for the Volume-method is limited to the number of pallets per partner. For the other cost allocation methods, we need to know the total transportation cost for all coalitions (the Nucleolus and the Shapley value), or all coalitions that contain $|N| - 1$ partners (the ECM).

The most prevalent cost allocation methods in co-operative game theory are the Shapley value and the Nucleolus. Both methods have a lot of the desired properties (Moulin, 1988), but they approach fairness in a different way. The Shapley value is utilitarian as it only considers a player's cooperative productivity when determining its share of the gain (Loehman & Whinston, 1974), whereas the Nucleolus is egalitarian as it divides the benefits so that the excesses are as equal as possible. When choosing a proper allocation method, it should be decided which principles and characteristics are regarded the most fair in the situation at hand. As shown in Table 2, a cost allocation method that possesses all listed characteristics does not exit. In the next section we therefore focus on finding a cost allocation method that specifically is adequate for rewarding *flexibility*.

Table 2
Cost allocation methods and their different properties.

	Shapley	Nucleolus	Volume	ECM	ABC
Pareto-efficiency[a]	X	X	X	X	X
Individual Rationality[b]	X	X	–	X	–
Anonimity[c]	X	X		X	–
Stability[d]	–	X	–	–	–
Coalitional monotonicity[e]	X	–	X	X	X
Population monotonicity[f]	–	X	X	–	–
Dummy property[g]	X	X	–	–	–
Additivity [h]	X	–	–	–	–
Decentralizability[i]	–	–	X	X	–

[a] $\sum_{i \in N} \varphi = c(N)$, the exact total cost should be allocated among the players.
[b] $\varphi_i \leqslant c(i)$, a player should not be allocated a cost that is higher than its stand-alone cost.
[c] $c(S \cup i) = c(S \cup j) \rightarrow \varphi_i = \varphi_j$. Players that are the same (generate the same cost in each coalition), should be allocated the same cost.
[d] $\sum_{i \in S} \varphi_i \leqslant c(S)$ and $\sum_{i \in N} \varphi_i = c(N)$, a player should not be able to do better in a coalition S.
[e] $v(S) \leqslant v'(S)$ and $v(T) = v'(T) \forall T \neq S \Rightarrow \varphi_i \leqslant \varphi'_i$, if the value of a coalition rises (drops) while the value of all the other coalitions remains the same, all the players in that coalition should receive a higher (lower) share.
[f] $\forall i \in N$, if $M = N \cup j$ and $v(N) = v(M) \Rightarrow \varphi_i^N \geqslant \varphi_i^M$, if no new surplus opportunities arise when a new player joins, none of the players should benefit.
[g] $v(S \cup i) = 0 \ \forall S \Rightarrow \varphi_i = 0$, if a player adds zero benefits to a coalition, it should not receive a share of the coalition gain.
[h] $\varphi(i + j) = \varphi(i) + \varphi(j)$, the cost allocation cannot be influenced by making larger coalitions in advance.
[i] A player's share depends only on its own cost/benefit and the aggregated cost/benefit of the coalition.

3. The importance of flexibility and cost allocation mechanisms in creating coalition gains

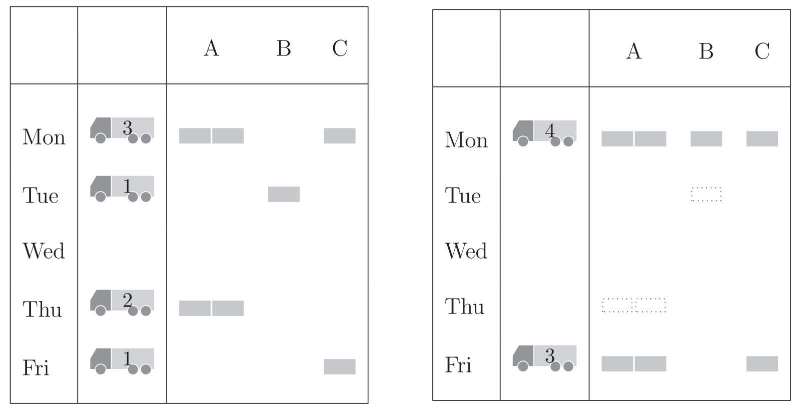
Optimization opportunities in transportation increase with the number of deliveries to perform, and with the flexibility of delivery terms. In general, the more restrictions companies put on their operational planning (i.e., the more strict their delivery terms are), the higher the total operational cost will be. Note that we only consider transportation costs. Flexibility may incur other (often hidden) costs such as inventory costs.

In the hypothetical example in Fig. 1, company A, B and C deliver to the same region. Company A delivers two pallets on Monday and Thursday, company B delivers a pallet on Tuesday and company C has two deliveries on Monday and Friday. If all companies work individually, five shipments are required. However, in case they collaborate, a reduction can be achieved from five shipments to four (Fig. 1a). When company A and B allow their shipments to be moved one day, the shipments can be synchronized and a further reduction is realized to two shipments (Fig. 1b).

In many situations, a coalition can decrease its distribution cost when companies allow synchronization and adapt their delivery dates to those of their partners. The more flexible the time windows of the deliveries, the better they can be combined with the deliveries of the partners.

In general, flexible delivery terms contribute to a decrease of the total distribution cost of the coalition. Therefore, flexibility of the partners with respect to their delivery terms has to be encouraged. As a coalition consists of individual companies acting out of self-interest, the allocation of the total cost can play an important role in the promotion of flexibility, and therefore a factor that cannot be underestimated when planning the distribution of the orders of the coalition.

Assuming that any player in the coalition can take on two positions, flexible or rigid, a cost allocation method should possess the following characteristics in order to maintain the incentives to collaborate and to be flexible:



(a) The number of trips when A, B and C collaborate but are not flexible

(b) The number of trips when A, B and C collaborate, A and B allow their orders to be moved one day

Fig. 1. Example of the gains that can be achieved by collaborating and being flexible.

- A company in a coalition will never pay more than its (flexible) stand-alone cost.
- The behaviour of the other players remaining equal, a company will never pay more if it assumes a flexible position.
- A company that contributes more to the coalition gain when adopting a flexible position will receive a larger incentive to be flexible.

As seen in Fig. 1, company A and company B both contribute to the consolidation gain by allowing their orders to be moved one day. When either of them decides to relax its constraints, cost can be further reduced in the grand coalition as well as in the subcoalitions as shown in Table 3. Although company A does not postpone its order from Monday to Tuesday, it will do so when it is faced with a rigid company B. In Fig. 2, we show that when one of them decides to relax its constraints, all cost allocation methods reduce the cost allocated to the flexible company. The amount of the cost reduction, or the difference in allocated cost when partner i is rigid $\varphi_{\text{rigid},i}$ and when that partner is flexible $\varphi_{\text{flexible},i}$ is the size of the incentive that partner i receives to be flexible.

In Fig. 2, each cost allocation method attributes a lower cost to a partner when this partner relaxes its constraints then when he remains rigid. However, it is clear that cost allocation methods that use the *cost effect* or the *marginal cost* to allocate costs give a high incentive to be more flexible. This is logical, as being flexible has an immediate impact on the marginal cost of a player. However, it can be seen from Fig. 2 that, although the ECM provides the largest incentive to be flexible, it does not guarantee that a partner will receive an allocated cost that is lower than its stand-alone cost. The Shapley value on the other hand, also based on marginal costs, *guarantees* individual rationality (see Table 2).

Although Fig. 2 shows that ABC and the Volume-method allocate lower costs to flexible partners, they seem to be negatively biased towards large companies. In Fig. 2 the allocated cost by ABC and the Volume-method to the larger company A is always higher than the cost allocated by the other methods. When company A is not flexible, the allocated cost even exceeds the stand-alone cost. Moreover, the incentive to be flexible that ABC and the Volume-method give to the large partner are among the lowest of all the cost allocation methods. Although the incentive to be flexible by the Volume-method is still significant for a large company, this is mainly due to the fixed (too) large share in the cost that this company has to contribute. For a small company, the share is very small, and thus, the incentive to be flexible is almost non-existent.

The final cost allocation method, the Nucleolus, does not give incentives that are as large as the ECM or the Shapley value. However, it gives consistently significant incentives to be flexible, to large companies as well as to small ones. Moreover, as can be derived from Table 2, this cost allocation method guarantees individual rationality as well.

Even in this simple example, the different cost allocation methods yield different results. Cost allocation methods that use the number of pallets as their cost driver (ABC and the Volume-method) are more disadvantageous to larger companies than the other cost allocation methods. This is not surprising, as the number of pallets is in those cases mainly considered as the cost driver, and they do not consider the opportunities to achieve economies of scale. Basing the allocation only on the resulting costs on the other hand provides a cost allocation that is generally better for large companies, but more importantly, that provides a better, consistent incentive to be flexible. We have shown that using only the marginal cost of a player in the final coalition can be detrimental to the incentive to collaborate. In Frisk et al. (2010), more cost allocation methods allocating the separable cost or marginal cost entirely to the partner that has caused that cost, but offering more complex rules (based on stand-alone costs and possibly the costs of the subcoalitions) have been suggested. However, the analysis of these methods is outside the scope of this paper. We have shown that it is possible with a number of cost allocation methods, but preferably the Shapley value, to give incentives to be more flexible. However, we will show in Section 4.1 that there are still issues when there is an asymmetric flexibility effort in a two-partner coalition.

Table 3
Costs (in number of trips) of all (sub)coalitions for Fig. 1 for different flexibility scenarios.

	All rigid	A is flexible	B is flexible
$c(A)$	2	2	2
$c(B)$	1	1	1
$c(C)$	2	2	2
$c(AB)$	3	2	2
$c(AC)$	3	2	3
$c(BC)$	3	3	2
$c(ABC)$	4	3	3

4. Rewarding flexibility in a coalition with two partners

4.1. Problem statement

In Section 3 we have rewarded the contribution of a partner given a set of orders and a set of constraints on the operational

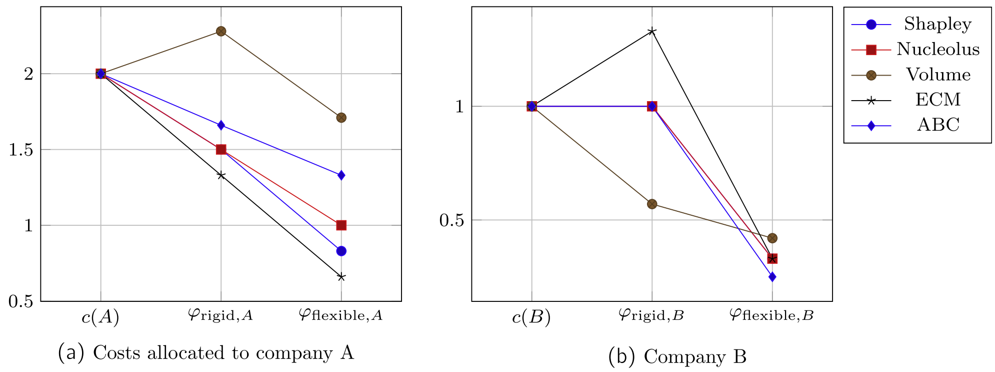


Fig. 2. Comparison of cost allocations for example in Fig. 1.

planning (e.g., delivery dates). This set is their baseline, i.e., when working alone, they organize their planning according to these constraints. Because of additional consolidation gains they could acquire, companies may decide to relax the constraints they impose.

However, not all changes to a company's delivery terms are without costs. Changing a fixed delivery day of a loyal customer, e.g., might incur some decrease in the goodwill of this customer towards the company. Other examples are changing warehouse location, using new types of pallets, other IT-systems, etc., all of which might be required when engaging in a logistic coalition. If a partner in the coalition feels that the changes required from it are larger than the changes required from the other partners, and does not see a significant difference in the share of the gains allocated to it in this situation, this partner might become dissatisfied and leave the coalition.

This problem can be alleviated by clearly studying and negotiating which changes require additional effort. The cost allocation methods listed in Section 2 however, will not consider effort when allocating cost, as only the measurable cost reduction is rewarded. The Shapley value specifically will only reward a company for making an effort to relax its constraints if this contributes to a decrease of the cost of either the grand coalition or one of the coalitions in which this company occurs in the Shapley formula. However, if this increased effort incurs additional costs, the company performing the change will want to be compensated. Identifying and quantifying the cost of relaxing a company's constraints is however often problematic, as this cost is not visible to the other partners, and often not even to the company itself. Adding an extra (arbitrary) flexibility cost to the total coalition cost is thus often not possible.

Kalai and Samet (1987) argue that in case of two partners, the Shapley allocation can easily be considered unfair. As a consequence of the Shapley formula, the consolidation gain in such a situation is always divided equally among the partners. This is shown in Appendix A. Moreover, it is proven by Aumann and Maschler (1985) that this is also the case for the Nucleolus. Finally, in Appendix B, we show that the ECM will give the same results. This makes sense given the fact that none of the coalition gains can be achieved without both of the companies being present in the coalition. However, even though the coalition gain might be divided in two, the effort delivered by both companies to create the coalition gain is not necessarily equal. Kalai and Samet (1987) propose to use a *weighted* Shapley value, which differs from the Shapley value as the latter assumes *symmetry*. This means that gains or costs that have additionally been realized in a certain coalition (thus, these costs or gains have not occured in any of the subsets of that coalition), are divided equally among the partners of that coalition. The weighted Shapley value on the other hand allows to define weights to distribute these gains or costs proportionally. Different ways to calculate the weighted Shapley value are described in Kalai and Samet (1987).

A problem with the approach of Kalai and Samet (1987) is that no recommendation on the values of the weights is given. In Soons (2011), the authors describe a situation in which four inland terminal operators have negotiated a fixed time window, and the weighted Shapley value is used to allocate waiting time. The weights are based on the barging volumes, assuming that a higher volume increases the negotiation power. Still, allowing to define weights allows room for manipulation.

As no clear cost can be associated to the additional flexibility, nor a unique set of weights, we propose a different method to divide the coalition costs. In our method, the contribution of a partner that *relaxes* its terms of deliveries is explicitly incorporated. In this way, the difference between a partner that adopts a "rigid" position with respect to its delivery terms and that same partner that adopts a "flexible" position, can be measured and adequately rewarded.

Consider the example in Fig. 1. Assume that company B has withdrawn from the coalition and company A and C continue to collaborate. To increase the consolidation gains, company A allows its orders to be synchronized. It is however impossible to know, knowing only $c(A) = 2, c(C) = 2$ and $c(AC) = 2$, which company has allowed to synchronize its orders. The allocated cost to company A and company C can be found in Fig. 3, showing that the gains are distributed evenly by the ECM, the Shapley value and the Nucleolus, and that the Volume-method and ABC even discriminate against company A.

4.2. Incorporation of rigid situations

In our alternative approach, we propose to incorporate the situations where a flexible partner remains rigid. In this manner, it is possible to detect which partner has the highest impact on the total cost of the coalition by becoming flexible. This is done by assuming that there exists a rigid partner i^r, and a possibility for this partner to relax his constraints i^f. In other words, when partner i^r is added to a coalition, the costs that partner i would cause

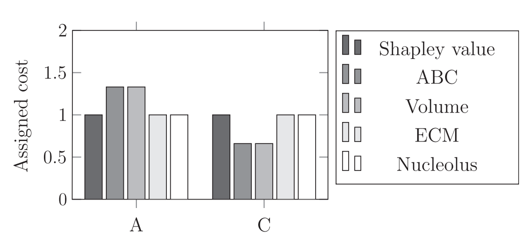


Fig. 3. Cost allocated to company A and C when working together in a two-partner coalition.

when it does not make changes to its deliveries are considered. When i^f is part of a coalition that also includes i^r, it gives the coalition the opportunity to alter deliveries from player i to its advantage. Of course, adding player i^f to a coalition that does not include player i^r does not have any effect ($\forall S \subseteq N \setminus i^r, i^f : c(S \cup i^f) = c(S)$). Moreover, adding player i^f (i.e., the option for player i to be flexible) to a coalition (containing player i^r or not), will never increase the cost of this coalition, as deliveries will not be altered if this would not benefit the coalition ($c(S \cup i^f) \leqslant c(S)$). Table 4 summarizes the new notation and characteristic function. Because each player i now consists out of two "hypothetical" players, the number of subcoalitions (or different situations) has increased from $2^{|N|}$ to $2^{2|N|}$.

Applied to the example of Section 4.1, we split partner A and C each in two hypothetical players A^r and A^f, and C^r and C^f. Consequently, additional information is needed, which can be found in Table 5.

The cost that will be paid by company A will be the cost related to partner A^r and the cost related to A^f. When reformulating a cost allocation method using additional, rigid situations, we use the symbol $\varphi^{costallocationmethod,\rho}$. We show in Sections 4.3–4.5 that the allocation $\varphi^{Shapley,\rho}$, $\varphi^{Nucleolus,\rho}$ and $\varphi^{ECM,\rho}$ will differ from the original approach. We exclude ABC and the Volume-method, as those cost allocation methods do not use the costs of different subcoalitions to calculate the allocation.

4.3. The Shapley value incorporating rigid and flexible situations in a two-partner coalition

Given a two-partner coalition $\{ij\}$, the allocated cost to i is:

$$\begin{aligned}
\varphi_i^{Shapley,\rho} &= \varphi_{i^r}^{Shapley} + \varphi_{i^f}^{Shapley} \\
&= \frac{1}{4}c(i^r) + \frac{1}{12}[c(i^r i^f) - c(i^f)] + \frac{1}{12}[c(i^r j^r) - c(j^r)] \\
&\quad + \frac{1}{12}[c(i^r j^f) - (j^f)] + \frac{1}{12}[c(i^r i^f j^r) - c(i^f j^r)] + \frac{1}{12}[c(i^r j^r j^f) \\
&\quad - c(j^r j^f)] + \frac{1}{12}[c(i^r i^f j^f) - c(i^f j^f)] + \frac{1}{4}[c(i^r i^f j^r j^f) - c(i^f j^r j^f)] \\
&\quad + \frac{1}{4}c(i^f) + \frac{1}{12}[c(i^r i^f) - c(i^r)] + \frac{1}{12}[c(i^f j^r) - c(j^r)] \\
&\quad + \frac{1}{12}[c(i^f j^f) - c(j^f)] + \frac{1}{12}[c(i^r i^f j^r) - c(i^r j^r)] + \frac{1}{12}[c(i^f j^r j^f) \\
&\quad - c(j^r j^f)] + \frac{1}{12}[c(i^r i^f j^f) - c(i^r j^f)] + \frac{1}{4}[c(i^r i^f j^r j^f) - c(i^r j^r j^f)]
\end{aligned}$$

Given that the option of flexibility will have no effects on the costs of a coalition S if the rigid partner is not a part of that coalition, or,

$$\forall S \setminus i^r : c(S \cup i^f = c(S))$$

and

$$c(i^f) = c(j^f) = 0$$

Table 4
Notation and the characteristic function for the approach incorporating rigid situations.

Notation	
i^r	A rigid partner i
i^f	The possibility for partner i to relax its constraints
N'	The new grand coalition, incorporating i^r and i^f
Characteristic function	
$\lvert N' \rvert = 2\lvert N \rvert$	
$c(i^f) = 0$	
$\forall S \subseteq N' \setminus i^r, i^f : c(S \cup i^f) = c(S)$	
$c(S \cup i^f) \leqslant c(S)$	

Table 5
All subcoalitions and related costs for the new approach incorporating rigid situations.

$c(A^r) = 2$	$c(A^r A^f) = 2$	$c(A^f C^f) = 0$	$c(A^r C^r C^f) = 3$
$c(A^f) = 0$	$c(A^r C^r) = 3$	$c(C^r C^f) = 2$	$c(A^f C^r C^f) = 2$
$c(C^r) = 2$	$c(A^r C^f) = 2$	$c(A^r A^f C^r) = 2$	$c(A^r A^f C^r C^f) = 2$
$c(C^f) = 0$	$c(A^f C^r) = 2$	$c(A^r A^f C^f) = 2$	

The allocated cost to partner i according to our alternative method is

$$\begin{aligned}
\varphi_i^{Shapley,\rho} &= \frac{1}{6}c(i^r) + \frac{1}{3}c(i^r i^f) - \frac{1}{6}c(j^r) + \frac{1}{6}c(i^r i^f j^r) - \frac{1}{6}c(i^r j^r j^f) \\
&\quad - \frac{1}{3}c(j^r j^f) + \frac{1}{2}c(i^r i^f j^r j^f)
\end{aligned}$$

This new formula ensures that partner i will be allocated a higher cost than in the old approach if the consolidation gain when partner i is flexible and j is rigid is smaller than when the roles are reversed. In other words, it gives a higher incentive to be flexible to the partner that can create the highest consolidation gains by becoming flexible.

$$\begin{aligned}
\varphi_i^{Shapley,\rho} > \varphi_i^{Shapley} &\iff \frac{1}{6}c(i^r) + \frac{1}{3}c(i^r i^f) - \frac{1}{6}c(j^r) + \frac{1}{6}c(i^r i^f j^r) \\
&\quad - \frac{1}{6}c(i^r j^r j^f) - \frac{1}{3}c(j^r j^f) + \frac{1}{2}c(i^r i^f j^r j^f) \\
&\quad > \frac{1}{2}c(i^r i^f j^r j^f) - \frac{1}{2}c(j^r j^f) + \frac{1}{2}c(i^r i^f) \\
&\iff c(i^r) + 2c(i^r i^f) - c(j^r) + c(i^r i^f j^r) \\
&\quad - c(i^r j^r j^f) - 2c(j^r j^f) > -3c(j^r j^f) + 3c(i^r i^f) \\
&\iff c(i^r i^f j^r) - c(j^r) - c(i^r i^f) > c(i^r j^r j^f) - c(j^r j^f) \\
&\quad - c(i^r) \iff c(i^r i^f) + c(j^r) - c(i^r i^f j^r) \\
&\quad < c(j^r j^f) + c(i^r) - c(i^r j^r j^f)
\end{aligned}$$

There is however a problem calculating the Shapley value using this alternative approach. As can be derived from Table 2, the Shapley value guarantees individual rationality. Translating individual rationality to this alternative setting however, implies the following:

$$\varphi_i^{\rho} \leqslant c(i^r i^f)$$

A partner i no longer consists out of one player, but is defined as a subcoalition. As the Shapley value does not guarantee stability, in other words, does not guarantee that the allocated cost to a subcoalition is not greater than the cost of that specific subcoalition, we can no longer guarantee individual rationality by using this alternative approach.

However, we show that such an "irrational" allocation will only occur for a partner i when the difference between the consolidation gains that can be achieved by i being flexible (and j is not) and the consolidation gains when j is flexible (and i is not) is extremely large to the extent that it triples the consolidation gain when both are flexible.

$$\begin{aligned}
\varphi_i^{Shapley,\rho} > c(i^r i^f) &\iff \frac{1}{6}c(i^r) + \frac{1}{3}c(i^r i^f) - \frac{1}{6}c(j^r) + \frac{1}{6}c(i^r i^f j^r) \\
&\quad - \frac{1}{6}c(i^r j^r j^f) - \frac{1}{3}c(j^r j^f) + \frac{1}{2}c(i^r i^f j^r j^f) > c(i^r i^f) \\
&\iff c(i^r) - c(j^r) + c(i^r i^f j^r) - c(i^r j^r j^f) - 2c(j^r j^f) \\
&\quad + 3c(i^r i^f j^r j^f) > 4c(i^r i^f) \iff 3c(i^r i^f j^r j^f) \\
&\quad - 3c(i^r i^f) - 3c(j^r j^f) - c(i^r i^f) - c(j^r + c(i^r i^f j^r) \\
&\quad > -c(j^r j^f) - c(i^r) + c(i^r j^r j^f) \\
&\iff -3[(c(i^r i^f) + c(j^r j^f) - c(i^r i^f j^r j^f)] \\
&\quad + [c(i^r i^f) + c(j^r - c(i^r i^f j^r)] \\
&\quad < [c(j^r j^f) + c(i^r) - c(i^r j^r j^f)]
\end{aligned}$$

In other words, when the consolidation gains are very small, and there is a large difference between the consolidation gains that are observed when one partner is flexible and the other remains rigid, it is possible that a partner is allocated a cost that will force him out of the coalition. However, it remains a question whether, when considering the argument that we aim to reward flexibility that has a (invisible) cost, a partner that can only decrease costs to a very small extent by becoming flexible, will actually choose a flexible profile in the final coalition. When this partner chooses to be flexible, questions can be raised whether there is an actual cost linked to that flexibility in the first place. It might be better to regard this flexible profile as the starting point of partner *i*, and assume that partner *i* does not have a flexible option.

4.4. The ECM incorporating rigid and flexible situations in a two-partner coalition

When transforming the ECM to our alternative approach, we assume that each "hypothetical" player will pay the marginal costs it causes. The remainder is divided equally among all the players.

The cost attributed to player *i* is thus:

$$\begin{aligned}\varphi_i^{ECM,\rho} &= \varphi_{i^r}^{ECM} + \varphi_{i^f}^{ECM}\\ &= [c(i^r i^f j^r j^f) - c(j^r j^f)] + [c(i^r i^f j^r j^f) - c(i^r j^r j^f)]\\ &\quad + \frac{2}{4}\Big\{c(i^r i^f j^r j^f) - [c(i^r i^f j^r j^f) - c(j^r j^f) + c(i^r i^f j^r j^f) - c(i^r j^r j^f)\\ &\quad + c(i^r i^f j^r j^f) - c(i^r i^f) + c(i^r i^f j^r j^f) - c(i^r i^f j^r)]\Big\}\\ &= \frac{1}{2}[c(i^r i^f j^r j^f) + c(i^r i^f j^r) + c(i^r i^f) - c(i^r j^r j^f) - c(j^r j^f)]\end{aligned}$$

As it also incorporates the costs of the collaboration effects when one partner remains rigid, the allocated cost will differ from the traditional ECM when the cost generated in $c(i^r i^f j^r)$ and $c(i^r j^r j^f)$ are different from each other. We show that partner *i* will be allocated a higher cost when the cost of a subcoalition containing a flexible *i* and a rigid *j* is higher than the cost of a subcoalition containing a rigid *i* and a flexible *j*.

$$\begin{aligned}\varphi_i^{ECM,\rho} > \varphi_i^{ECM} &\iff \frac{1}{2}[c(i^r i^f j^r j^f) + c(i^r i^f j^r) + c(i^r i^f) - c(i^r j^r j^f) - c(j^r j^f)]\\ &\qquad > \frac{1}{2}c(i^r i^f j^r j^f) - \frac{1}{2}c(j^r j^f) + \frac{1}{2}c(i^r i^f)\\ &\iff c(i^r i^f j^r) > c(i^r j^r j^f)\end{aligned}$$

The traditional ECM does not guarantee individual rationality (see Table 2) and neither does the alternative method of calculating the ECM. It can easily be derived that when the difference in costs for partners having different flexibility profiles is larger than the consolidation gain, the alternative ECM will allocate a cost that will not be accepted by one of the players.

$$\begin{aligned}&\varphi_i^{ECM,\rho} > c(i^r i^f)\\ &c(i^r i^f j^r) - c(i^r j^r j^f) > c(i^r i^f) + c(j^r j^f) - c(i^r j^r j^f)\end{aligned}$$

4.5. The Nucleolus incorporating rigid and flexible situations in a two-partner coalition

As the Nucleolus attempts to find an allocation that is in the centre of all the possible stable allocations, incorporating additional situations will enforce an allocation that differs from the original allocation by the Nucleolus. As additional subcoalitions imply that the core becomes more constrained, excluding allocations for which $\sum_{k\in\{i^r i^f\}} x_k < v(i^r i^f)$, $\sum_{k\in\{j^r j^f\}} x_k < v(j^r j^f)$, $\sum_{k\in\{i^r j^r j^f\}} x_k < v(i^r j^r j^f)$ and $\sum_{k\in\{i^r i^f j^r\}} x_k < v(i^r i^f j^r)$, more allocations will become non-viable options and the core will become smaller. Moreover, the smaller set of stable profit allocations will contain a higher share of allocations that benefit the additional subcoalitions for which $v(S)$ is the highest. As the Nucleolus will choose the centre of this set, the partner that proves to create the largest benefits when becoming flexible, will gain in comparison to the original method. Moulin (1988) proves that for three-partner coalitions, in which the Nucleolus will favour the partners of *strong* subcoalitions, i.e. there exist two subcoalitions S and T for which the sum $v(S) + v(T)$ is larger than the consolidation gain $v(N)$. The Nucleolus will bring twice as much inequality between surplus shares as the Shapley value. In general, when the core is small or non-existent, the cost allocation by the Nucleolus will favour the flexible partner significantly in comparison to the allocation by the Shapley value.

As the Nucleolus will find a stable allocation when the core is non-empty, the individual rationality of this alternative method is guaranteed.

4.6. Experiments

Applying the new formulas on the example as stated in Section 4.1, we find that all three methods favour the flexible company A (see Fig. 4).

To further test the alternative calculation methods, an additional experiment has been set up, assuming slightly more complex collaborations. In this experiment, goods of collaborating partners are gathered in a central warehouse, and distributed to their clients by creating round trips. We assume that a partner can be flexible, i.e., allow its orders to be delivered one day later. To be able to calculate the costs of the different subcoalitions, we use an algorithm that can solve a *capacitated periodic vehicle routing problem*. The algorithm creates efficient round trips for each given day, allowing certain orders to occupy routes one day later. The full results of the experiment and a description of the algorithm can be found at http://antor.ua.ac.be/Downloads.

For each partner, the *capacitated multi-period vehicle routing problem* is solved twice: once allowing orders to be moved, once remaining a rigid position. Secondly, the problem is solved for both partners combined. Again, we assume different flexibility profiles for both partners. A summary of the results of these simulations can be found in Appendix C in Table 6.

The allocated costs are calculated without incorporating rigid situations. (As mentioned in Section 4.1, this solution is equal for either the Shapley value, the ECM or the Nucleolus.) However, when incorporating the different rigid situations, the allocations will differ from each other. These results can be found in Table 7.

Our experiments show that the alternative methods generally give better incentives for flexibility. The ECM however, has allocated a cost that is higher than the stand-alone cost in two out of twelve instances. In these instances, the difference in costs of the subcoalitions where one partner is flexible and the other is not is larger than the consolidation profit. It should be noted that the ECM in general does not guarantee a rational allocation.

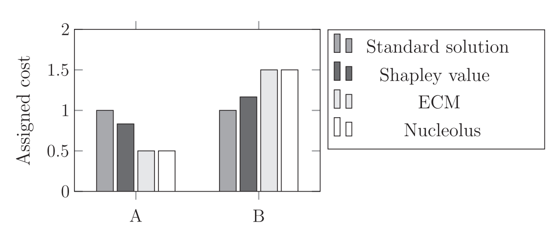


Fig. 4. Allocations for company A and C when using the alternative calculation methods.

The Shapley value, although it does not guarantee individual rationality, either has allocated costs that generate profits for both partners. As stated in Section 4.3, a cost allocation that exceeds the stand-alone cost by the adapted Shapley formula will be more rare.

The Nucleolus always guarantees stability, and will thus never allocate a cost that is higher than the stand-alone cost of a partner. Although the Shapley value and the Nucleolus both allocate rational allocations, the examples show that the allocated costs by the Nucleolus are more similar to the allocation by the ECM than the Shapley value. Both reward the flexible partner with the highest impact to a greater extent than the Shapley value. The main reason for this high reward being respectively the large emphasis on the final marginal cost, and the small core. As stated in Section 4.5, the Nucleolus favours the most flexible partner to a significantly larger extent when the consolidation gains in the subcoalitions are very large in comparison to the consolidation gain.

In our generated instances, in terms of the *profit*, the combination of a rigid and a flexible partner is often the most attractive option. The Nucleolus will give great importance to the fact that the partner with the highest flexibility impact prefers a combination where the other partner is rigid, and will therefore give a large part of the gain to satisfy that partner (and thus, a very small part to the other partner). However, one should be aware that the highest profit does not correspond to the lowest *cost*. It is thus important to evaluate beforehand to which extent the incentive to be flexible for the partner with the highest flexibility impact should surpass the incentive to collaborate for the other partner.

5. Conclusions

In this paper, we have argued that flexibility is a crucial issue in horizontal logistic collaborations. Not only can an increased flexibility with respect to the partners' delivery terms decrease the total coalition cost. Additionally, a company that adopts a flexible position can also see its own cost reduced, provided that a proper allocation method is used. Although several cost allocation methods have been proposed in the (game theory) literature, none of these explicitly reward an increased flexibility. The choice of a fair allocation method is already difficult when not integrating flexibility. As fairness is subjective, different properties of fairness have been defined in literature.

We have evaluated different cost allocation methods, and arrive at the conclusion that cost allocation methods that base their allocation principle on the costs generated in all or some of the subcoalitions (the Shapley value, the ECM and the Nucleolus) give the most consistent, fair incentives for flexibility. However, to reward the value of flexibility in small coalitions, given that effort is asymmetric and flexibility does not come without a cost, we propose alternative ways to calculate these allocation methods, incorporating the rigid profiles as well.

All three cost allocation methods show to be able to reward the most flexible partner. However, the ECM can still easily attribute a cost that is higher than the stand-alone cost of a partner. The Nucleolus on the other hand guarantees individual rationality. The Nucleolus should however be carefully applied when the profit generated by the partner with the highest flexibility impact is very large compared to the total consolidation gain. The partner that has the lowest impact when flexible will receive a very small share of the profit, giving that partner only a small incentive to collaborate.

Contrary to the Nucleolus, The Shapley value cannot guarantee individual rationality. We have however argued that such an 'irrational' allocation will occur only when an incorrect form of flexibility is encouraged. One should therefore always evaluate beforehand whether the right flexibility is encouraged. Our alternative calculation methods are only useful when it is believed that flexibility will have a negative effect on the partners (e.g. an increase in stock, a reduction in goodwill of the clients, an increased risk of late deliveries, etc.) and the effort is asymmetric.

In conclusion, when dividing costs in a coalition with asymmetric effort, cost allocation methods such as the ECM, the Shapley value and the Nucleolus suffice. Each of them however have drawbacks, an important being that the ECM does not guarantee individual rationality. In a two-partner coalition however, the gain is always divided equally. Considering that flexibility often comes with a cost, this can easily be considered unfair by the partners. Our alternative approach when calculating the ECM, the Shapley value or the Nucleolus can give solace. However, one should consider beforehand the extent to which the incentive to be flexible can surpass the incentive to collaborate, whether the right flexibility is encouraged and whether there is a danger to allocate a cost that will exceed the stand-alone cost. Although this new approach still needs to be used with care and careful negotiation beforehand, and partners still need to select the correct cost allocation method, it has the main advantage that the share of profit for a partner, that has been achieved by becoming flexible, is not chosen arbitrarily, but is based on the cost effects of that partner becoming flexible.

Appendix A. The standard solution of the Shapley value for a two-partner coalition

Given $N = i, j,\ |N| = 2$, and $c(\emptyset) = 0$:

$$\begin{aligned}\varphi_i^{Shapley} &= \frac{1}{2}[c(ij) - c(j)] + \frac{1}{2}[c(i) - (c(\emptyset)] \\ &= \frac{1}{2}c(ij) - \frac{1}{2}c(j) + c(i) - \frac{1}{2}c(i) \\ &= c(i) - \frac{1}{2}[c(ij) - c(i) - c(j)] = c(i) - \frac{1}{2}\nu(12) \\ \varphi_j^{Shapley} &= \frac{1}{2}[c(ij) - c(i)] + \frac{1}{2}[c(i) - c(\emptyset)] = c(j) - \frac{1}{2}\nu(12)\end{aligned}$$

Table 6
Cost and profit function.

$c(A^r)$	$c(B^r)$	$c(A^rA^f)$	$c(B^rB^f)$	$c(A^rB^r)$	$c(A^rB^rB^f)$	$c(A^rA^fB^r)$	$c(A^rA^fB^rB^f)$	$\nu(A^rB^r)$	$\nu(A^rB^rB^f)$	$\nu(A^rA^fB^r)$	$\nu(A^rA^fB^rB^f)$
2312.51	3710.06	1715.37	2393.91	5083.32	3560.19	3990.54	2549.56	939.25	1146.23	715.88	1559.72
7978.18	2442.49	6527.79	2442.49	9462.46	9389.59	7946.06	7918.87	958.21	1031.08	2474.61	1051.41
2340.59	3000.43	1473.39	2762.13	4214.21	3707.83	4214.01	3623.36	1126.81	1394.89	888.71	612.16
2242.16	1989.19	2242.16	1989.19	4231.35	4173.51	4231.35	3919.29	0	57.84	0	312.06
2726.01	6913.43	1536.5	5073.76	7566.31	6067.38	7082.8	5938.62	2073.13	1732.39	716.97	671.64
5764.69	14386.53	4174.11	13592.43	18212.28	17024.41	17209.28	16478.44	1938.94	2332.71	2147.84	1288.1
2746.82	9471.66	2695.7	8429.3	11787.74	9633.28	10676	9419.18	430.74	1542.84	500.12	1705.82
8670.13	10146.66	6455.06	8828.93	16527.68	14709.52	15335.14	14515.65	2289.11	2789.54	2163.92	768.34
6706.5	5081.68	5382.05	3212.9	9388.73	8116.09	7563.95	7063.88	2399.45	1803.31	2355.45	1531.07
2676.07	7875.93	2033.15	6507.96	9886.45	7932.65	8286.58	7512.51	665.55	1251.38	897.45	1028.6
9038.8	6041.39	7131.19	4781.92	13044.49	12357.27	11532.22	11064.37	2035.7	1463.45	2288.5	848.74
13761.16	3120.66	12527.69	2119.94	14835.42	14775.87	14139.75	13963.44	2046.4	1105.23	1741.35	684.19

Table 7
The distribution by the Shapley value, the ECM and the Nucleolus when allocating using rigid situations.

φ_A	φ_B	$\varphi_A^{Shapley,\rho}$	$\varphi_B^{Shapley,\rho}$	$\varphi_A^{Shapley,\rho} - c(A^rA^f)$	$\varphi_B^{Shapley,\rho} - c(B^rB^f)$	$\varphi_A^{ECM,\rho}$	$\varphi_B^{ECM,\rho}$	$\varphi_A^{ECM,\rho} - c(A^rA^f)$	$\varphi_B^{ECM,\rho} - c(B^rB^f)$	$\varphi_A^{Nuc,\rho}$	$\varphi_B^{Nuc,\rho}$	$\varphi_A^{Nuc,\rho} - c(A^rA^f)$	$\varphi_B^{Nuc,\rho} - c(B^rB^f)$
935.51	1614.05	887.4	1662.16	827.97	731.75	1150.685	1398.875	564.685	995.035	1.210	1.340	505	1.054
6002.085	1916.785	6003.228	1915.641667	524.5617	526.8483333	5280.32	2638.55	1247.47	−196.06	5.504	2.415	1.024	27
1167.31	2456.05	1356.49	2266.87	116.9	495.26	1420.4	2202.96	52.99	559.17	1.431	2.192	42	570
2086.13	1833.16	2095.77	1823.52	146.39	165.67	2115.05	1804.24	127.11	184.95	2.086	1.833	156	156
1200.68	4737.94	1261.557	4677.063333	274.9433	396.6966667	1708.39	4230.23	−171.89	843.53	1.472	4.467	64	607
3530.06	12948.38	3693.618	12784.82167	480.4917	807.6083333	3622.495	12855.95	551.615	736.485	3.901	12.577	273	1.015
1842.79	7576.39	1851.37	7567.81	844.33	861.49	2364.15	7055.03	331.55	1374.27	2.533	6.886	163	1.543
6070.89	8444.76	6324.717	8190.933333	130.3433	637.9966667	6383.7	8131.95	71.36	696.98	6.358	8.158	97	671
4616.515	2447.365	4433.77	2630.11	948.28	582.79	4340.445	2723.435	1041.605	489.465	5.132	1.932	250	1.281
1518.85	5993.66	1456.997	6055.513333	576.1533	452.4466667	1695.815	5816.695	337.335	691.265	1.823	5.689	210	819
6706.82	4357.55	6677.335	4387.035	453.855	394.885	6294.295	4770.075	836.895	11.845	6.516	4.548	615	234
12185.6	1777.845	12118.37	1845.073333	409.3233	274.8666667	11867.54	2095.905	660.155	24.035	11.932	2.032	596	88

Appendix B. The standard solution of the ECM for a two-partner coalition

$$\varphi_i^{ECM} = c(ij) - c(j) + \frac{1}{2}[c(ij) - (c(ij) - c(i))]$$
$$= c(ij) - c(j) + c(i) - \frac{1}{2}c(i) = c(i) - \frac{1}{2}v(12)$$
$$\varphi_j^{ECM} = c(ij) - c(i) + \frac{1}{2}[c(ij) - (c(ij) - c(j))] = c(j) - \frac{1}{2}v(12)$$

Appendix C. Results of the experiments run in Section 4.6

See Tables 6 and 7.

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