Analysis of different cost allocation methods in a collaborative transport setting

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Collaborative transport
How does it differ from 3PL bundling?

- Both responsible for creating benefits
  - Has to be rewarded
- Sharing the risks and the benefits
  - equal share? (small vs. large volume)
Methodology

- Choose set of cost allocation methods
- Test them on various scenarios
  - Different number of partners
  - Partners differ in size (order size, #orders)
  - Partners differ in time window sizes

**Objective**

How do these cost allocation react in when partners are not equal?
Cost allocation methods

Used in practice

- Stand-alone (SA)
  \[ w_i = \frac{c(i)}{\sum_{j \in N} c(j)} \]
  \[ x_i = w_i c(N) \]

- Volume
Cost allocation methods

Used in game-theory

- Shapley

\[ x_i = \sum_{S \subseteq N \setminus i} \frac{|S|!(|N| - |S| - 1)!}{|N|!} \times (c(S \cup i) - c(S)) \]
**Nucleolus**

Maximize smallest excess \( e(x; S) = \sum_{i \in S} v_i - v(S) \)
Cost allocation methods

- **Equal Charge Method (ECM)**
  \[ m_i = c(N) - c(N \setminus i) \]
  \[ x_i = m_i + \frac{c(N) - \sum j m_j}{|N|} \]

- **Alternative Cost Avoided Method (ACAM)**
  \[ x_i = m_i + (c(N) - \sum j m_j) \frac{c(i) - m_i}{\sum j c(j) - m_j} \]

- **Cost Gap Method**
  \[ a_i = \min_{S \mid i \in S} (c(S) - \sum_{j \in S} m_j) \]
Cost allocation methods

- Equal profit method (EPM)

\[
\begin{align*}
\min \quad & f \\
\text{s.t.} \quad & f \geq \frac{x_i}{c(i)} - \frac{x_j}{c(j)} \quad \forall i, j \\
& \sum_{j \in S} x_j \leq c(S) \quad \forall S \subseteq N \\
& \sum_{j \in N} x_j = c(N) - \sum_{i \in N} c(i) \quad \forall x_i \geq 0, \quad \forall i \\
& x_i \geq 0 \quad \forall i
\end{align*}
\]
Results: Number of partners

- The smaller the number of partners, the smaller the difference between cost allocation methods.
- For two partners, none of the allocation methods are adequate to incentivize flexibility.
Results: Total number of pallets

Allocated cost to company B (4073 pallets), collaborating with partners A (967 pallets) and C (1375 pallets)
Results: Total number of pallets

- Do not use the Volume method
- Dividing cost using stand-alone cost (SA): least benefit for large companies
- Dividing profit using stand-alone cost (EPM): most benefit for large companies
- Shapley, ACAM, Nucleolus and CGM uses stand-alone cost + costs of subgroups
  - Shapley, ACAM: slight benefit for small companies
  - Nucleolus, CGM: slight benefit for large companies
Results: Flexible time windows

Allocated cost to company A (967 pallets), collaborating with partners B (4073 pallets) and C (1375 pallets) · 10^4
Results: Flexible time windows

- Based on stand-alone costs (EPM, SA): incentives to lower own cost, not collaborative costs
- Marginal costs (ECM, CGM, ACAM, Shapley) or all the costs of the subcoalitions (Nucleolus)
Strong subcoalitions

Most of the benefit can already be created with a subset of the partners $S \in N$ and $T \in N$ for which $\nu(S) + \nu(T) \geq \nu(N)$

- Stability becomes important
- Nucleolus, EPM
Conclusion

Different number of pallets?

Different flexibility?

Strong subcoalitions?

How many partners?

Yes

No

Yes

No
Conclusion

- There is no best method
  - Dependent on type of coalition
- A difficult method is not always necessary