Secure Vehicle Routing
Models and algorithms to increase security and reduce costs in the cash-in-transit sector

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Introduction & motivations

Models & algorithms to limit the route risk
- RCVRP
- RCVRPTW
- MO-RCVRP

Risk mitigation by route unpredictability
- kd-VRP

Conclusions & future work
**Context → Cash-in-transit sector (CIT)**

- Worldwide cash transactions ≈ $11.6 trillion (+1.75% between 2008-2012)
- 79% of large retailers in the USA rely on specialized carriers for cash deposits
- Worldwide cost of handling cash > $300 billion per year
Risk → Crime is a real threat

- The U.K. presents the highest risk rate and loss records in Europe:
  - $\approx £500$ billion transported each year ($£1.4$ billion per day)
  - More than 1,000 attacks against cash carriers in the UK in 2008
  - $\approx £150$ million losses in the last five years

- With the downturn in the global economy, crime is on the rise:
  - February 2013 Foggia Italy: Robbery of 300 Kg of gold
  - February 2013 Brussels Belgium: Brussels Airport diamond heist
  - March 2014 Cagliari Italy: Robbery of €6 million
Introduction
Introduction

Risk Identification & Analysis

↓

Risk Measurement & Modelling

↓

Risk Mitigation
Introduction

Risk Identification & Analysis

⇒

Foreseeable vs Unforeseeable Consequences

Risk Measurement & Modelling

↓

Risk Mitigation
Introduction

- Risk Identification & Analysis
  - Foreseeable vs Unforeseeable Consequences
- Risk Measurement & Modelling
  - Risk Index & Risk Awareness
- Risk Mitigation
Introduction

Risk Identification & Analysis

⇒

Foreseeable vs Unforeseeable Consequences

↓

Risk Measurement & Modelling

⇒

Risk Index & Risk Awareness

↓

Risk Mitigation

⇒

Preventative Measures & Secure Routing Planning
Risk mitigation
Risk mitigation
Risk mitigation

Route risk limitation
  ↓
  RcvRP
  RcvRPtw
  Mo-RcvRP

Route unpredictability
  ↓
  kd-VRP
Risk index

- Risk index ∝ \{ 
  - Exposure of the vehicle outside of the depot
  - Loss of transported valuables (foreseeable consequences)
\}

- The *global route risk* $R_e^r$ is the risk incurred by the vehicle along route $r$ upon its return to the depot.

**Current Values**

- **CC** = 1 k€
- **TT** = 10 km
- **RE** = $0 \cdot 10 = 0$

---

CC = Cash Carried   TT = Travel Time   RE = Risk Exposure
Risk index

- Risk index \( \propto \) \{ Exposure of the vehicle outside of the depot, Loss of transported valuables (foreseeable consequences) \}

- The global route risk \( R_e^r \) is the risk incurred by the vehicle along route \( r \) upon its return to the depot

Current Values

- CC = 3 k€
- TT = 18 km
- RE = 1 \cdot 8 = 8

CC = Cash Carried  TT = Travel Time  RE = Risk Exposure
**Risk index**

- Risk index $\propto \left\{ \begin{array}{l} 
\text{Exposure of the vehicle outside of the depot} \\
\text{Loss of transported valuables (foreseeable consequences)} 
\end{array} \right.$

- The *global route risk* $R_e^r$ is the risk incurred by the vehicle along route $r$ upon its return to the depot.

### Current Values

- $CC = 8$ k€
- $TT = 28$ km
- $RE = 1 \cdot 8 + 3 \cdot 10 = 38$

**Abbreviations:**

- CC = Cash Carried
- TT = Travel Time
- RE = Risk Exposure
Risk index

- Risk index $\propto \begin{cases} 
\text{Exposure of the vehicle outside of the depot} \\
\text{Loss of transported valuables (foreseeable consequences)}
\end{cases}$

- The global route risk $R_r^e$ is the risk incurred by the vehicle along route $r$ upon its return to the depot

Current Values
- CC = 8 k€
- TT = 41 km
- RE = $1 \cdot 8 + 3 \cdot 10 + 8 \cdot 13 = 142$

CC = Cash Carried  TT = Travel Time  RE = Risk Exposure
Risk Constrained Vehicle Routing Problem (RcVRP)*

Determine a set of routes (a route is a tour that begins at a depot s, traverses a subset of the customers in a specified sequence and returns to a depot e) where:

- Each customer must be assigned to exactly one of the routes
- The total risk for each route must not exceed the risk threshold
- The routes should be chosen to minimize the total travel cost

**Risk threshold** → Maximum level of risk that a CIT firm accepts to face

gement with **Insurance companies**!

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Eight metaheuristics have been developed and coded in JAVA:

**GRASP**

- m-Cwg: Multi-start Clarke and Wright heuristic with greedy randomized selection mechanism
- m-Nng: Multi-start Nearest neighbour with greedy randomized selection mechanism
- m-Tnng: Multi-start TSP nearest neighbour with greedy randomized selection mechanism plus splitting

**ILS**

- p-Cwg: Perturbation Clarke and Wright heuristic with greedy randomized selection mechanism
- p-Nng: Perturbation Nearest neighbour with greedy randomized selection mechanism
- p-Tnng: Perturbation TSP nearest neighbour with greedy randomized selection mechanism plus splitting
- p-Tlk: Perturbation TSP Lin-Kerningan plus splitting
- aLNS: Ant colony optimization with Large Neighbourhood Search
Overview of the metaheuristics

**m-Cwg**

1. Start
2. Initialize Metaheuristic
3. Find an initial solution using CWg
4. Intensification by VND
5. Is CS better than BS?
   - Yes: Update the BS
   - No: Perturb the CS
   - No: Is the # of restart reached?
     - Yes: Exit
     - No: Update the BS

**p-Cwg**

1. Start
2. Initialize Metaheuristic
3. Find an initial solution using CWg
4. Intensification by VND
5. Perturb the CS
6. Is CS better than BS?
   - Yes: Update the BS
   - No: Is the # of restart reached?
     - Yes: Exit
     - No: Update the BS
Overview of the metaheuristics

**p-TLK**

1. Initialize Metaheuristic
2. Solve the beneath Tsp using LK
3. Find an initial solution splitting the Tsp tour
4. Perturb the CS
5. Intensification by VND
6. Is CS better than BS?
7. Is the # of restart reached?
   - yes: Update the BS
   - no: Perturb the CS
8. Exit

**aLNS**

1. Initialize Metaheuristic
2. Solve the beneath Tsp using ACO
3. Find an initial solution splitting the Tsp tour
4. Perturb the CS
5. Intensification by VND
6. Max iter no impr. reached?
   - yes: Exit
   - no: Is the # of restart reached?
7. Is CS better than BS?
   - yes: Update the BS
   - no: Perturb the CS
Test instances

The RcvRP has not been studied before → no available test instance

- **SET R** → 180 small instances (9 Basic Instances × 5 Risk Levels × 4 Demand vectors)
- **SET V** → 55 medium-large instances (11 Basic Instances × 5 Risk Levels)
- **SET O** → 32 medium-large instances with known optimal solutions
- **SET S** → 32 medium-large instances with known optimal solutions
### Quality & robustness of the initial RcvRP solutions

<table>
<thead>
<tr>
<th>Metaheuristic</th>
<th>Best % gap</th>
<th>Average % gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-Tlk</td>
<td>2.39%</td>
<td>2.39%</td>
</tr>
<tr>
<td>m-Cwg (p-Cwg)</td>
<td>3.16%</td>
<td>4.57%</td>
</tr>
<tr>
<td>aLNS</td>
<td>5.26%</td>
<td>7.73%</td>
</tr>
<tr>
<td>m-TNng (p-TNng)</td>
<td>8.72%</td>
<td>12.26%</td>
</tr>
<tr>
<td>m-Nng (p-Nng)</td>
<td>9.52%</td>
<td>13.68%</td>
</tr>
</tbody>
</table>

### Results for all benchmark sets

#### (a) Best GAP

<table>
<thead>
<tr>
<th>Test Set</th>
<th>m-Cwg</th>
<th>m-Nng</th>
<th>p-Cwg</th>
<th>p-Nng</th>
<th>m-TNng</th>
<th>p-TNng</th>
<th>p-Tlk</th>
<th>aLNS</th>
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</thead>
<tbody>
<tr>
<td>SET R</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
<td>0.00%</td>
</tr>
<tr>
<td>SET V</td>
<td>0.52%</td>
<td>1.88%</td>
<td>1.11%</td>
<td>1.46%</td>
<td>1.55%</td>
<td>1.32%</td>
<td>0.86%</td>
<td>0.50%</td>
</tr>
<tr>
<td>SET O</td>
<td>4.91%</td>
<td>6.35%</td>
<td>5.21%</td>
<td>5.88%</td>
<td>5.83%</td>
<td>5.57%</td>
<td>4.25%</td>
<td>3.92%</td>
</tr>
<tr>
<td>SET S</td>
<td>3.48%</td>
<td>4.90%</td>
<td>3.95%</td>
<td>4.78%</td>
<td>4.64%</td>
<td>3.97%</td>
<td>3.20%</td>
<td>2.91%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>2.23%</strong></td>
<td><strong>3.28%</strong></td>
<td><strong>2.57%</strong></td>
<td><strong>3.03%</strong></td>
<td><strong>3.01%</strong></td>
<td><strong>2.71%</strong></td>
<td><strong>2.08%</strong></td>
<td><strong>1.83%</strong></td>
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</tbody>
</table>

#### (b) Avg. GAP

<table>
<thead>
<tr>
<th>Test Set</th>
<th>m-Cwg</th>
<th>m-Nng</th>
<th>p-Cwg</th>
<th>p-Nng</th>
<th>m-TNng</th>
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<tbody>
<tr>
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<td>0.10%</td>
<td>1.28%</td>
<td>0.43%</td>
<td>0.88%</td>
<td>0.58%</td>
<td>0.69%</td>
<td>0.09%</td>
<td>0.10%</td>
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<tr>
<td>SET V</td>
<td>1.38%</td>
<td>3.75%</td>
<td>2.51%</td>
<td>3.38%</td>
<td>2.84%</td>
<td>2.92%</td>
<td>1.89%</td>
<td>1.44%</td>
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<tr>
<td>SET O</td>
<td>5.29%</td>
<td>7.29%</td>
<td>5.63%</td>
<td>6.65%</td>
<td>6.49%</td>
<td>6.22%</td>
<td>4.57%</td>
<td>4.54%</td>
</tr>
<tr>
<td>SET S</td>
<td>4.26%</td>
<td>5.93%</td>
<td>4.62%</td>
<td>5.81%</td>
<td>5.57%</td>
<td>4.80%</td>
<td>3.86%</td>
<td>3.75%</td>
</tr>
<tr>
<td><strong>Average</strong></td>
<td><strong>2.76%</strong></td>
<td><strong>4.57%</strong></td>
<td><strong>3.30%</strong></td>
<td><strong>4.18%</strong></td>
<td><strong>3.87%</strong></td>
<td><strong>3.66%</strong></td>
<td><strong>2.60%</strong></td>
<td><strong>2.46%</strong></td>
</tr>
</tbody>
</table>

- Intensification & diversification phases more effective in aLNS
- aLNS and p-Tlk find the highest # of optimal solutions for SET O and SET S
Risk Constrained Vehicle Routing Problem with Time Window constraints (RCVRPTW)*

Determine $k$ vehicle routes where:

- Each customer must be assigned to exactly one of the $k$ routes
- The total risk for each route must not exceed the risk threshold $T$
- The beginning of the service must fall inside the customer’s hard time window and no waiting times are allowed
- The routes should be chosen to minimize total travel cost

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Metaheuristics

**GRASP**  $\Rightarrow$  **m-Grc-VND**: Multi-start greedy randomized Constructive heuristic with variable neighbourhood descend

**ILS**  $\Rightarrow$  **p-Grc-VND**: Perturb-and-improve greedy randomized Constructive heuristic with variable neighbourhood descend

- 24 CVRPTW instances from *Cordeau et al. (2001)* and *Homberger et al. (1999)* adapted for the Rcvrptw

$\rho - \phi$ vs. % gap between the best obtained solutions and the best known CVRPTW solutions

Relation between $\rho - \phi$ and average CPU time
Mo-RcVRP description

Multi-objective Risk Constrained Vehicle Routing Problem (Mo-RcVRP)*

Determine a set of vehicle routes where:

▶ A vehicle cannot transport more than a maximum amount of valuables (capacity constraint)
▶ Each customer must be assigned to exactly one of the routes
▶ The routes should be chosen to minimize total travel cost and the highest global route risk at the same time

\[ \text{Mo-RcVRP} \Rightarrow \begin{cases} \text{Multi-objective extension of the RcVRP} \\ \text{Multiple depots included} \end{cases} \]

Worst case scenario

Total risk exposure $f_2(x)$

- $\max_{r=\{1,2,3\}} R^r_e = 150$

Route risk values

- $R^1_e = 142$
- $R^2_e = 136$
- $R^3_e = 150$
PmOO-ILS metaheuristic

- Multi-Objective Metaheuristic (МОМН) ⇒ Pareto frontier

\[ f_1(x) \]

\[ f_2(x) \]
PMOO-ILS metaheuristic

- Multi-Objective Metaheuristic (Момн) $\Rightarrow$ Pareto frontier

![Diagram showing Pareto frontier and dominated region with markers for Pareto frontier approximation and dominated points.](image)
PMOO-ILS metaheuristic

- Multi-Objective Metaheuristic (MOMH) ⇒ Pareto frontier
- Multi-Criteria Decision Making (MCDM) ⇒ Solution selection
PMOO-ILS metaheuristic

- Multi-Objective Metaheuristic (MOMH) ⇒ Pareto frontier
- Multi-Criteria Decision Making (MCDM) ⇒ Solution selection

$P/m.o.sc/o.sc = M/o.sc/m.sc + M/c.d.m/m.sc$

Progressive multi-objective optimization

$P/moo=MoMh+Mcdm$

- Pareto frontier approximation
- Pareto frontier
- Dominated region
- “Best” solution
Phase 1: Populate the solution archive → ILS with 7 Lso
Pmoo-ILS metaheuristic

- **Phase 1:** Populate the solution archive $\rightarrow$ ILS with 7 Lso
- **Phase 2:** Update the solution archive
  - PROMETHEE II to rank solutions and find a new search direction
  - ILS to generate a new solution in that direction

$$H_j(d_j) = \begin{cases} 
0, & d_j \geq q_j \\
\frac{d_j - q_j}{p_j - q_j}, & p_j < d_j < q_j \\
1, & d_j \leq p_j 
\end{cases}$$
Phase 1: Populate the solution archive → ILS with 7 Lso
Phase 2: Update the solution archive
  ▶ PROMETHEE II to rank solutions and find a new search direction
  ▶ ILS to generate a new solution in that direction
Phase 3: Update the solution archive
Test instances

- **Set L** ⇒ 33 multi-depot VRP instances designed by Cordeau et al. (1997) with 48-360 nodes and 2-9 depots
- **Set P** ⇒ 5 realistic large study cases from 100 to 1,035 nodes and 4 depots

<table>
<thead>
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<th>id</th>
<th>Category</th>
<th>#</th>
<th>Demand [LB; UB]</th>
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<td>56</td>
<td>[10; 400]</td>
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<td>2</td>
<td>Clothes shops</td>
<td>295</td>
<td>[5; 100]</td>
</tr>
<tr>
<td>3</td>
<td>Supermarkets</td>
<td>324</td>
<td>[5; 150]</td>
</tr>
<tr>
<td>4</td>
<td>Shopping-centres</td>
<td>48</td>
<td>[20; 400]</td>
</tr>
<tr>
<td>5</td>
<td>Jewellery stores</td>
<td>312</td>
<td>[5; 200]</td>
</tr>
<tr>
<td></td>
<td><strong>Total</strong></td>
<td></td>
<td><strong>1035</strong></td>
</tr>
</tbody>
</table>
Computational results

Instance *p12* in Set L

Instance *Mo-700* in Set P

Instance *Mo-500* in Set P

Instance *Mo-1039* in Set P
In many European countries, cash-in-transit companies must by law determine different routes for each vehicle in order to:

- Easily change its plans in case of unforeseen circumstances (accidents, road works, etc.)
- Increase security by making the vehicle routes more unpredictable

The same vehicle route cannot be used more than two consecutive times.
kd-VRP description

k-dissimilar Vehicle Routing Problem (kd-VRP)*

Given a similarity metric, a feasible solution to the kd-VRP is a set of $k$ feasible VRP solutions for which the difference between each pair of alternative solutions is larger than a certain threshold

- Each alternative solution must obey the traditional VRP constraints
- The objective of the kd-VRP is to minimize the cost (total distance) of the worst alternative solution in the set

In some real-life applications the constraint imposing no shared edges between the alternative solutions might be too stringent!

The index to measure similarities between VRP solutions → literature on shortest path (Akgün et al., 2000 and Vanhove, 2012)
kd-VRP formulation

Master Problem

\[
\min \sum_{i} \max_{y_i} f(y_i) \\
\forall i \in \{1, \ldots, k\} \\
\delta(y_i, y_j) \leq T_s \quad \forall i, j \in \{1, \ldots, k\}; i \neq j \\
y_i \in \Omega \quad \forall i \in \{1, \ldots, k\}
\]

Slave Problem

\[
\min \sum_{h} \sum_{(i,j) \in E} c_{ij} x_{ij}^h \\
\sum_{j \in V \setminus \{0\}} x_{0j}^h = \sum_{j \in V \setminus \{0\}} x_{j0}^h = 1 \quad \forall h \in N \\
\sum_{h \in N} \sum_{i \in V} x_{ij}^h = \sum_{h \in N} \sum_{j \in V} x_{ji}^h = 1 \quad \forall i \in V \setminus \{0\} \\
\sum_{i \in V} \sum_{j \in V \setminus \{0\}} d_{ij} x_{ij}^h \leq C \quad \forall h \in N \\
\sum_{h \in N} \sum_{i \in Q} \sum_{j \notin Q} x_{ij}^h \geq 1 \quad \forall Q \subset V; Q \neq \emptyset \\
x_{ij}^h \in \{0, 1\} \quad \forall (i, j) \in E; \forall h \in N
\]

kd-VRP key controls

- \( k \) \( \Longrightarrow \) Number of alternative solutions to be generated
- \( T_s \) \( \Longrightarrow \) Maximum similarity threshold
Solution approach

- **Iterative Penalty Method (IPM\_kd)**
  - ILS → Find an alternative solution using the current cost matrix
  - Penalization function → Update the cost matrix forcing the algorithm to explore a different area of the search space

- 51 instances from the VRP library

<table>
<thead>
<tr>
<th>Author</th>
<th># Instances</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>Augerat et al</td>
<td>8</td>
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<td>Christofides et al</td>
<td>7</td>
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<td>Fisher</td>
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<td>13</td>
<td>76-386</td>
</tr>
<tr>
<td>Golden et al.</td>
<td>20</td>
<td>201-484</td>
</tr>
</tbody>
</table>

- Given the similarity measure, an instance of the VRP is transformed in an instance of the RCVRP by adding only two parameters: \( k \) and \( T_s \)
Computational results

- Tuning of the *heuristic parameters* → Impact of the *kd-VRP* parameters on the solution quality
Conclusions

▶ We addressed some risk management activities which can be carried out during the routing planning stage

▶ Little economic investments are required → Risk mitigation and robbery prevention affordable also for small-sized cash carriers!

▶ The main contributions of this thesis are:
  ▶ Or to manage risk in the cash-in-transit sector
  ▶ Introduction of new unexplored variants of the VRP by implicitly considering risk for cash-in-transit applications
  ▶ Generation of new sets of instances which are publicly available
  ▶ Efficient metaheuristics to solve the proposed models
Future research

- Real applications involving C1T firms

- Several extensions of the proposed models by considering:
  - route length restrictions
  - delivery of money to customers
  - precedence/priority constraints

- Extension of the models to other sectors where risk and cost represent two critical issues
Thanks for your attention!

“It always seems impossible until it is done”

Nelson Mandela