New Metaheuristics for the Risk Constraint Cash-in-Transit Vehicle Routing Problem

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Introduction

Risk Constraint

Problem Formulation
  Strategies Description
  Metaheuristics Components

Experimental Results

Future work
Context $\implies$ Cash and valuables in transportation sector

Risk $\implies$ Crime is a real challenge

- In the U.K. alone, there is an estimated £500 billion transported each year (£1.4 billion per day);
- In 2008, there were 1,000 documented attacks against CIT couriers in the UK;
- With the downturn in the global economy, CIT crime is on the rise;
- Money stolen in CIT attacks is a major source of funding for organized crime.
Risk Constraint

RISK

Probability that the unwanted event occurs \((P)\) \(\Rightarrow\) Exposure of the vehicle outside of the depot

\(\times\)

Consequences of the unwanted event \((M)\) \(\Rightarrow\) Loss of the money or valuables transported

\[ R = P \cdot M \] (1)
RISK

Probability that the unwanted event occurs (\( P \)) \( \Rightarrow \) Exposure of the vehicle outside of the depot

\[ R = P \cdot M \]  \hspace{1cm} (1)

\[ \times \]

Consequences of the unwanted event (\( M \)) \( \Rightarrow \) Loss of the money or valuables transported

Risk threshold!
Assuming that a vehicle only picks up cash along its route a risk index can be defined as follows:

\[ R_j^r = R_i^r + P_i^r \cdot d_{ij} \quad (2) \]

where \( P_i^r \) is the amount of money on board of the vehicle when it arrives at node \( i \) along route \( r \) and \( d_{ij} \) is the length of the arc \((i, j)\).

- The index is a cumulative (increasing) measure of the risk incurred by the vehicle while it travels along its route.
- The *global route risk* \((GR^r)\) is the risk incurred by the vehicle upon its return to the depot.
- \( GR^r \) is limited to a certain maximum value \( T \) (*risk threshold*).
Example Risk Constraint

Risk Threshold
- $T=150 \text{ km} \cdot \text{€}$

Current Values
- $CC=1 \text{ €}$
- $TT=10 \text{ km}$
- $RE=0 \cdot 10 = 0 \text{ km} \cdot \text{€}$

CC=Cash Carried
TT=Travel Time
RE=Risk Exposure
Example Risk Constraint

Risk Threshold
- $T = 150 \text{ km} \cdot \text{€}$

Current Values
- $CC = 3 \text{ €}$
- $TT = 18 \text{ km}$
- $RE = 1 \cdot 8 = 8 \text{ km} \cdot \text{€}$

CC = Cash Carried
TT = Travel Time
RE = Risk Exposure
Example Risk Constraint

Risk Threshold
- $T = 150 \text{ km} \cdot \text{€}$

Current Values
- $CC = 8 \text{ €}$
- $TT = 28 \text{ km}$
- $RE = 1 \cdot 8 + 3 \cdot 10 = 38 \text{ km} \cdot \text{€}$

CC = Cash Carried
TT = Travel Time
RE = Risk Exposure
Example Risk Constraint

Risk Threshold
- $T = 150 \text{ km} \cdot \text{€}$

Current Values
- $CC = 8 \text{ €}$
- $TT = 41 \text{ km}$
- $RE = 1 \cdot 8 + 3 \cdot 10 + 8 \cdot 13 = 142 \text{ km} \cdot \text{€}$

CC = Cash Carried
TT = Travel Time
RE = Risk Exposure
Example Risk Constraint

Risk Threshold
- $T = 150 \text{ km} \cdot \text{€}

Current Values
- $CC = 4 \text{ €}$
- $TT = 12 \text{ km}$
- $RE = 0 \cdot 12 = 0 \text{ km} \cdot \text{€}$

$CC = \text{Cash Carried}$
$TT = \text{Travel Time}$
$RE = \text{Risk Exposure}$
Example Risk Constraint

Risk Threshold
- $T = 150 \text{ km} \cdot \text{€}$

Current Values
- $CC = 9 \text{ €}$
- $TT = 19 \text{ km}$
- $RE = 4 \cdot 7 = 28 \text{ km} \cdot \text{€}$
Example Risk Constraint

Risk Threshold

- $T = 150 \text{ km} \cdot \text{€}$

Current Values

- $CC = 9 \text{ €}$
- $TT = 32 \text{ km}$
- $RE = 4 \cdot 7 + 9 \cdot 13 = 145 \text{ km} \cdot \text{€}$

CC = Cash Carried
TT = Travel Time
RE = Risk Exposure
Risk-constrained Cash-in-Transit Vehicle Routing Problem

Determine a set of routes (a route is a tour that begins at the depot, traverses a subset of the customers in a specified sequence and returns to the depot) where:

▶ Each customer must be assigned to exactly one of the routes;
▶ The global route risk for each route must not exceed the risk threshold;
▶ The routes should be chosen to minimize total travel cost.

[Talarico, Sörensen and Springael in 2012]
Risk-constrained Cash-in-Transit Vehicle Routing Problem

Determine a set of routes (a route is a tour that begins at the depot, traverses a subset of the customers in a specified sequence and returns to the depot) where:

- Each customer must be assigned to exactly one of the routes;
- The global route risk for each route must not exceed the risk threshold;
- The routes should be chosen to minimize total travel cost.

[Talarico, Sørensen and Springael in 2012]

No single tour as a consequence of the risk threshold!
We were unable to solve problems with more than 10 nodes in a reasonable time using CPLEX (Instances with 16 nodes solved in \(\sim 15\) min)

In a previous work 7 metaheuristics have been developed:

- **m-CWg** and **p-CWg**: Modified Clarke and Wright with Grasp
- **m-NNg** and **p-NNg**: Nearest Neighborhood with Grasp
- **m-TNNg** and **p-TNNg**: TSP Nearest Neighborhood with Grasp plus Splitting
- **p-TLK**: TSP Lin Kernighan plus Splitting

In this study 2 new solution approaches are introduced:

- **m-ACS**: multi-start Ant Colony System plus Splitting
- **p-ACS**: perturb-and-improve Ant Colony System plus Splitting
Metaheuristics overview 1/2

- **Step 1:** Generate an initial solution for the RCTVRP
  - **Step 1.1:** Generate a big TSP tour
Metaheuristics overview 1/2

- **Step 1**: Generate an initial solution for the RCTVRP
  - Step 1.1: Generate a big TSP tour
  - Step 1.2: Split the TSP tour in feasible routes
Metaheuristics overview 2/2

- Step 2: Optimization using a VNS block
Metaheuristics overview 2/2

- **Step 2**: Optimization using a VNS block

- **Step 3**: Escaping local optima
Generate a big TSP tour

- Relaxation of the risk constraint
- Ant colony system (described in Dorigo and Gambardella 1996) for solving the underlying TSP problem
Variant of the splitting procedure described in Prins 2004.

An arc \((i - 1, j)\) is added in the auxiliary graph if the route visiting the node \(i\) to the node \(j\), in the order they appear in the giant tour (or in the reverse order), is feasible.

The best possible way to split the TSP tour in feasible routes, is achieved by finding the shortest path from node 0 to node \(n\) in the auxiliary graph.
The VNS block used in m-ACS and p-ACS is made by:

- A Reversion operator;
- 7 Local Search Operators:
  - Intra Route Local Search
    - Or-Opt Operator
    - Relocate Operator
    - Two-Opt Operator
  - Inter Route Local Search
    - Exchange Operator
    - Relocate Operator
    - Cross-Exchange Operator
    - Two-Opt Operator
Intra-Route Local Search Operators

**Or-Opt Operator**

**Two-Opt Operator**

**Relocate Operator**
Inter-Route Local Search Operators

Exchange Operator

Relocate Operator

Cross-Exchange Operator

Two-Opt Operator
Diversification Strategy 1/2

**Multi-start**

**Algorithm:** m-ACS

```plaintext
while (ρ is not reached) do
    Generate a solution \( \bar{x} \) for the TSP using the ACS heuristic;
    Find a feasible solution \( x \) for the RCTVRP applying the splitting procedure to \( \bar{x} \);
    Improve \( x \) using the VNS block;
end
Report best solution \( x^* \);
```

**Destroy-and-repair**

**Algorithm:** p-ACS

```plaintext
Generate a solution \( \bar{x} \) for the TSP using the ACS heuristic;
Find a feasible solution \( x \) for the RCTVRP applying the splitting procedure to \( \bar{x} \);
while (\( \varrho \) is not reached) do
    Improve \( x \) using the VNS block;
    Perturb \( x \);
repeat
    Destroy \( x \)
until (\( \xi \cdot m \) routes from \( x \) have been destroyed);
Repair \( x \);
end
Report best solution \( x^* \);
```
The *Total Score* (repair mechanism) is given by the sum of two partial scores representing the relative ranking of the unvisited nodes ordering them according to:

- Decreasing distance from the depot;
- Increasing demand value.

**Algorithm:** Repair mechanism

Initialise the parameters $\gamma_p$, $\gamma_d$ and $\alpha$;
Let $U$ be list of nodes which need to be reassigned to new routes;
Let $x$ be the current solution containing the non destroyed routes;
Create a new empty route to be added to $x$;

while $(U \neq 0)$ do

  Set the current node=depot;
  Assign a total score to the nodes in $U$ using $\gamma_p$ and $\gamma_d$;
  Order the nodes in $U$ following a decreasing order of the *total score*;
  Create a restricted list $RC_e$ containing the first $\alpha$ nodes in $U$;
  Use a GRASP selection to find the first node in the current route;
  Set the current node equal to the first node;
  Remove the current node from $U$;

  repeat
  Create a List of eligible nodes $L_e$ containing the nodes that can be reached from the current node respecting the risk constraint;
  if $(L_e \neq 0)$ then
  Use a GRASP selection to find the next node in the current route;
  Set the next node equal to the current node;
  Remove the current node from $U$;
  else
  Create a new empty route to be added to $x$;
  end
  until $(L_e \neq 0)$;

end
Report the solution $x$;
Metaheuristic Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho - \varrho )</td>
<td>Number of restarts (multi-start structure) / Number of perturbations (destroy-and-repair structure)</td>
<td>5, 100, 1000</td>
</tr>
<tr>
<td>( \xi )</td>
<td>Maximum percentage number of routes in the current solution to be destroyed</td>
<td>- 100%</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>Size of the restricted candidate list (repair heuristic)</td>
<td>- 3</td>
</tr>
<tr>
<td>( \gamma_d )</td>
<td>Coefficient used to weight the score for the distance (repair heuristic)</td>
<td>- 0.5</td>
</tr>
<tr>
<td>( \gamma_p )</td>
<td>Coefficient used to weight the score for the demand (repair heuristic)</td>
<td>- 0.5</td>
</tr>
<tr>
<td>ACS - Repetition</td>
<td>Number of initial giant tours to generate</td>
<td>3 6</td>
</tr>
<tr>
<td>Ants number</td>
<td>Number of ants (ACS heuristic)</td>
<td>8 16</td>
</tr>
<tr>
<td>Step number</td>
<td>Number of times each ant applies the transition rule and the local pheromone updating rule to incrementally build a giant tour (ACS heuristic)</td>
<td>3 6</td>
</tr>
</tbody>
</table>

- m-ACS and p-ACS have been coded in Java language.
- 30 trials for each configuration have been performed.
- Experiments performed on an Intel core i7-2760QM 2.40GHz processor with 4GB RAM.
Computational Results

- 64 instances for which the optimal solutions are known have been solved.
  - 32 instances in subset named set $O_s$;
  - 32 instances in subset named set $O_o$.

<table>
<thead>
<tr>
<th>SET $O_o$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$O_3$</td>
<td>$O_4$</td>
<td>$O_6$</td>
<td>$O_8$</td>
<td>$O_{16}$</td>
<td></td>
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<td><img src="" alt="Diagram" /></td>
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<td><img src="" alt="Diagram" /></td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>SET $O_s$</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_3$</td>
<td>$S_4$</td>
<td>$S_6$</td>
<td>$S_8$</td>
<td>$S_{16}$</td>
<td></td>
</tr>
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</tr>
</tbody>
</table>
Computational Results

- Percentage number of optimal solutions ($\rho$ and $\varrho$ equal to 100)

<table>
<thead>
<tr>
<th>Metaheuristic</th>
<th>set Oo</th>
<th>set Os</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-ACS</td>
<td>26%</td>
<td>21%</td>
<td>24%</td>
</tr>
<tr>
<td>m-ACS</td>
<td>6%</td>
<td>9%</td>
<td>7%</td>
</tr>
</tbody>
</table>

- Average percentage gap from optimal solutions ($\rho$ and $\varrho$ equal to 100)

<table>
<thead>
<tr>
<th>Metaheuristic</th>
<th>set Oo</th>
<th>set Os</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>p-ACS</td>
<td>8.7%</td>
<td>3.6%</td>
<td>6.2%</td>
</tr>
<tr>
<td>m-ACS</td>
<td>12.3%</td>
<td>5.3%</td>
<td>8.8%</td>
</tr>
</tbody>
</table>

Results for the 7 metaheuristics in their optimal setting (Previous work)

(a) Percentage number of optimal solutions

<table>
<thead>
<tr>
<th>Metaheuristic</th>
<th>set Oo</th>
<th>set Os</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>m-CWg</td>
<td>6%</td>
<td>0%</td>
<td>3%</td>
</tr>
<tr>
<td>m-NNg</td>
<td>6%</td>
<td>3%</td>
<td>4%</td>
</tr>
<tr>
<td>p-CWg</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
<tr>
<td>p-NNg</td>
<td>3%</td>
<td>3%</td>
<td>3%</td>
</tr>
<tr>
<td>m-TNNg</td>
<td>0%</td>
<td>21%</td>
<td>10%</td>
</tr>
<tr>
<td>p-TNNg</td>
<td>0%</td>
<td>18%</td>
<td>9%</td>
</tr>
<tr>
<td>p-TLK</td>
<td>35%</td>
<td>18%</td>
<td>26%</td>
</tr>
</tbody>
</table>

(b) Average percentage gap from optimal solutions

<table>
<thead>
<tr>
<th>Metaheuristic</th>
<th>set Oo</th>
<th>set Os</th>
<th>All</th>
</tr>
</thead>
<tbody>
<tr>
<td>m-CWg</td>
<td>16.5%</td>
<td>10.3%</td>
<td>13.4%</td>
</tr>
<tr>
<td>m-NNg</td>
<td>18.9%</td>
<td>13.3%</td>
<td>16.1%</td>
</tr>
<tr>
<td>p-CWg</td>
<td>22.2%</td>
<td>13.0%</td>
<td>17.6%</td>
</tr>
<tr>
<td>p-NNg</td>
<td>21.4%</td>
<td>15.2%</td>
<td>18.3%</td>
</tr>
<tr>
<td>m-TNNg</td>
<td>16.4%</td>
<td>5.7%</td>
<td>11.0%</td>
</tr>
<tr>
<td>p-TNNg</td>
<td>16.8%</td>
<td>6.2%</td>
<td>11.5%</td>
</tr>
<tr>
<td>p-TLK</td>
<td>11.6%</td>
<td>6.4%</td>
<td>9.0%</td>
</tr>
</tbody>
</table>
Future research

- Handling the risk as an additional objective to be minimized
- Finding some real applications of the problem involving CIT companies
- Several extensions of the problem can be proposed for future research taking in consideration some real-life constraints as:
  - route length restrictions;
  - time windows;
  - precedence relations.
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