Risk-constrained Cash-in-Transit Vehicle Routing Problem

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Summary

Introduction

Risk Constraint

Problem Formulation

Solution Strategies
  Strategies Description
  Metaheuristic Components

Experimental Results

Conclusions

Future works
Introduction

Context ⟷ Cash and valuables in transportation sector
Risk ⟷ Crime is a real challenge

- In the U.K. alone, there is an estimated £ 500 billion transported each year (£ 1.4 billion per day);
- In 2008, there were 1,000 documented attacks against CIT couriers in the UK;
- With the downturn in the global economy, CIT crime is on the rise;
- Money stolen in CIT attacks is a major source of funding for organized crime.
Risk Constraint

**RISK**

Probability that the unwanted event occurs $\times$ Exposure of the vehicle outside of the depot

$\times$

Consequences of the unwanted event $\times$ Loss of the money or valuables transported
Risk Constraint

**RISK**

- Probability that the unwanted event occurs
- Consequences of the unwanted event

⇒ Exposure of the vehicle outside of the depot
⇒ Loss of the money or valuables transported

**Risk threshold**
Example Risk Constraint

Risk Threshold
- $R = 150 \text{ km} \cdot \$$

Current Values
- $CC = 1 \$$
- $TT = 10 \text{ km}$$
- $RE = 0 \cdot 10 = 0 \text{ km} \cdot \$

$CC =$ cash carried  $TT =$ travel time  $RE =$ risk exposure
Example Risk Constraint

Risk Threshold
- \( R = 150 \text{ km} \cdot \$ \)

Current Values
- \( CC = 3 \$ \)
- \( TT = 18 \text{ km} \)
- \( RE = 1 \cdot 8 = 8 \text{ km} \cdot \$ \)

\( CC = \text{cash carried} \quad \text{TT} = \text{travel time} \quad \text{RE} = \text{risk exposure} \)
Example Risk Constraint

Risk Threshold
- $R=150 \text{ km} \cdot \$$

Current Values
- $CC=8 \text{ } \$
- $TT=28 \text{ km}$
- $RE=1 \cdot 8 + 3 \cdot 10 = 38 \text{ km} \cdot \$$

$CC=$ cash carried  $TT=$ travel time  $RE=$ risk exposure
Example Risk Constraint

Risk Threshold
- \( R = 150 \text{ km} \cdot \$ \)

Current Values
- \( CC = 8 \$ \)
- \( TT = 41 \text{ km} \)
- \( RE = 1 \cdot 8 + 3 \cdot 10 + 8 \cdot 13 = 142 \text{ km} \cdot \$ \)

\( CC = \) cash carried \( TT = \) travel time \( RE = \) risk exposure
Example Risk Constraint

Risk Threshold
- $R = 150 \text{ km} \cdot \$$

Current Values
- $CC = 4 \$ $$
- $TT = 12 \text{ km}$$
- $RE = 0 \cdot 12 = 0 \text{ km} \cdot \$

$CC = \text{cash carried} \quad TT = \text{travel time} \quad RE = \text{risk exposure}$
Example Risk Constraint

Risk Threshold
- \( R = 150 \text{ km} \cdot \$ \)

Current Values
- \( CC = 9 \text{ } \$ \)
- \( TT = 19 \text{ km} \)
- \( RE = 4 \cdot 7 = 28 \text{ km} \cdot \$ \)

CC = cash carried  TT = travel time  RE = risk exposure
Example Risk Constraint

Risk Threshold
- \( R = 150 \text{ km} \cdot \$ \)

Current Values
- \( CC = 9 \$ \)
- \( TT = 32 \text{ km} \)
- \( RE = 4 \cdot 7 + 9 \cdot 13 = 145 \text{ km} \cdot \$ \)

CC = cash carried  TT = travel time  RE = risk exposure
Example Risk Constraint

CC = cash carried  TT = travel time  RE = risk exposure

Risk Threshold
- \( R = 150 \text{ km} \cdot \$ \)

Current Values
- \( CC = 10 \$ \)
- \( TT = 15 \text{ km} \)
- \( RE = 0 \cdot 15 = 0 \text{ km} \cdot \$ \)
Example Risk Constraint

Risk Threshold
- \( R = 150 \text{ km} \cdot \text{\$} \)

Current Values
- \( CC = 10 \text{ \$} \)
- \( TT = 30 \text{ km} \)
- \( RE = 10 \cdot 15 = 150 \text{ km} \cdot \text{\$} \)

CC = cash carried  TT = travel time  RE = risk exposure
Risk-constrained Cash-in-Transit Vehicle Routing Problem

Determine a set of routes, where a route is a tour that begins at the depot, traverses a subset of the customers in a specified sequence and returns to the depot. Each customer must be assigned to exactly one of the routes. The total risk for each route must not exceed the risk threshold. The routes should be chosen to minimize total travel cost.

[Talarico, Sørensen and Springael in 2012]
RCTVRP Problem

Risk-constrained Cash-in-Transit Vehicle Routing Problem

Determine a set of routes, where a route is a tour that begins at the depot, traverses a subset of the customers in a specified sequence and returns to the depot. Each customer must be assigned to exactly one of the routes. The total risk for each route must not exceed the risk threshold. The routes should be chosen to minimize total travel cost.

[Talarico, Sørensen and Springael in 2012]

No single tour as a consequence of the risk threshold!
Problem Formulation

A necessary and sufficient condition on the RCTVRP feasibility is:

\[ T \geq \max_{i \in N} \{p_i \cdot t_{i0}\} \]

The condition guarantees that at least the problem has a feasible solution containing \( n \) routes.
Small instances of the RCTVRP have been solved to optimality using CPLEX

- The time needed to solve small instances of the problem increases exponentially;
- We were unable to solve problems with more than 10 nodes in a reasonable time (Instances with 16 nodes solved in \( \approx 15 \) min).

We need efficient Metaheuristics!
Seven Metaheuristics have been developed and coded in JAVA combining the following components:

- **4 Constructive Heuristics:**
  - Modified Clarke & Wright with Grasp (*Stochastic*);
  - Nearest Neighborhood with Grasp (*Stochastic*);
  - TSP Nearest Neighborhood with Grasp plus Splitting (*Stochastic*);
  - TSP Lin Kernighan plus Splitting (*Deterministic*).
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- **1 Local Search Block** (6 LS Operators);
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  - TSP Lin Kernighan plus Splitting (*Deterministic*).

- **1 Local Search Block** (6 LS Operators);

- **2 Structures:**
  - Multi-Start;
  - Perturbation.
Metaheuristics Overview

**MSMC&WG**

1. **Start**
2. Initialize Meta-Heuristic
3. Find an IS using MC&WG
4. Improve the CS using LSO
5. Is CS better than BS?
   - Yes: Update the BS
   - No: Perturb the CS
6. Is the # of restart reached?
   - Yes: Exit
   - No: Go back to 3

**MC&WGP**

1. **Start**
2. Initialize Meta-Heuristic
3. Find a CS using MC&WG
4. Improve the CS using LSO
5. Perturb the CS
6. Is CS better than BS?
   - Yes: Update the BS
   - No: Go back to 3
7. Is the # of restart reached?
   - Yes: Exit
   - No: Go back to 5
**Metaheuristics Overview**

**MSNNG**

- **Start**: Initialize Meta-Heuristic
- **Find an IS using NNG**
- **Improve the CS using LSO**
- **Is CS better than BS?**
  - **Yes**: Update the BS
  - **No**: Is the # of restart reached?
    - **Yes**: Exit
    - **No**: Is the # of restart reached?
      - **Yes**: Exit
      - **No**: Perturb the CS

**NNGP**

- **Start**: Initialize Meta-Heuristic
- **Find a CS using NNG**
- **Improve the CS using LSO**
- **Perturb the CS**
- **Is CS better than BS?**
  - **Yes**: Update the BS
  - **No**: Is the # of restart reached?
    - **Yes**: Exit
    - **No**: Is the # of restart reached?
      - **Yes**: Exit
      - **No**: Update the BS
Metaheuristics Overview

TSPLKSP

start

- Initialize Meta-Heuristic
- Solve the beneath TSP using LK
- Find a CS splitting the Big Tour
- Improve the CS using LSO
- Perturb the CS
- Is CS better than BS?
- Update the BS
- Is the # of restart reached?
- Exit

yes

no

no

yes
Modified Clarke & Wright Heuristic
Six different Local Search Operators have been used:

- **2 Intra-Route LSO**;
- **4 Inter-Route LSO**.
Intra-Route Local Search Operators

Two-opt

Or-opt
Inter-Route Search Operators

Two-opt

Relocate
Inter-Route Local Search Operators

Exchange

Cross Exchange
Since RCTVRP has not been studied before, no test instances are available in the literature.

- 55 Instance have been used to test the 7 Metaheuristics:
  - 11 Basic Instances taken from VRP lib;
  - 5 different Risk Constraint Levels.

<table>
<thead>
<tr>
<th>Name</th>
<th>Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>E022-04g</td>
<td>22</td>
</tr>
<tr>
<td>E026-08m</td>
<td>26</td>
</tr>
<tr>
<td>E030-03g</td>
<td>30</td>
</tr>
<tr>
<td>E036-11h</td>
<td>36</td>
</tr>
<tr>
<td>E045-04f</td>
<td>45</td>
</tr>
<tr>
<td>E051-05e</td>
<td>51</td>
</tr>
<tr>
<td>E072-04f</td>
<td>72</td>
</tr>
<tr>
<td>E101-08e</td>
<td>101</td>
</tr>
<tr>
<td>E121-07c</td>
<td>121</td>
</tr>
<tr>
<td>E135-07f</td>
<td>135</td>
</tr>
<tr>
<td>E151-12b</td>
<td>151</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Risk Level</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>RL1</td>
<td>$T = \max_{i \in N} {p_i \cdot t_{i0}}$</td>
</tr>
<tr>
<td>RL2</td>
<td>$1.5 \cdot RL1$</td>
</tr>
<tr>
<td>RL3</td>
<td>$2.0 \cdot RL1$</td>
</tr>
<tr>
<td>RL4</td>
<td>$2.5 \cdot RL1$</td>
</tr>
<tr>
<td>RL5</td>
<td>$3.0 \cdot RL1$</td>
</tr>
</tbody>
</table>
Two different kind of parameters have been analyzed:

- Instance characteristics;
- Metaheuristics parameters;

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
<th>Nr. of Levels</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nodes</td>
<td>22, 26, 30, 36, 45, 51, 72, 101, 121, 135, 151</td>
<td>11</td>
</tr>
<tr>
<td>Risk Level</td>
<td>1, 1.5, 2, 2.5, 3</td>
<td>5</td>
</tr>
<tr>
<td>Restart</td>
<td>1, 2, 3, 4, 5</td>
<td>5</td>
</tr>
<tr>
<td>TSP-Repetition</td>
<td>1, 2, 3, 4</td>
<td>4</td>
</tr>
<tr>
<td>Perturbation</td>
<td>0%, 5%, 10%, 15%, 20%, 25%</td>
<td>6</td>
</tr>
<tr>
<td>Or-Opt-Internal</td>
<td>on, off</td>
<td>2</td>
</tr>
<tr>
<td>Two-opt-Internal</td>
<td>on, off</td>
<td>2</td>
</tr>
<tr>
<td>Exchange</td>
<td>on, off</td>
<td>2</td>
</tr>
<tr>
<td>Relocate</td>
<td>on, off</td>
<td>2</td>
</tr>
<tr>
<td>Cross-Exchange</td>
<td>on, off</td>
<td>2</td>
</tr>
<tr>
<td>Two-opt-External</td>
<td>on, off</td>
<td>2</td>
</tr>
</tbody>
</table>
### Computational Results

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>Nodes</th>
<th>Computational Time</th>
<th>Instances with RL1</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
<td>MSNNG</td>
</tr>
<tr>
<td>0.25</td>
<td></td>
<td></td>
<td>MSMC&amp;WG</td>
</tr>
<tr>
<td>0.5</td>
<td></td>
<td></td>
<td>MC&amp;WGP</td>
</tr>
<tr>
<td>0.75</td>
<td></td>
<td></td>
<td>NNGP</td>
</tr>
<tr>
<td>1</td>
<td></td>
<td></td>
<td>MSTSPNNGS</td>
</tr>
<tr>
<td>1.25</td>
<td></td>
<td></td>
<td>TSPNNGSP</td>
</tr>
<tr>
<td>1.5</td>
<td></td>
<td></td>
<td>TSPLKSP</td>
</tr>
<tr>
<td>1.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.25</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.5</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.75</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Computational Time

**Instances with RL1**

- Time (s)
- Nodes
- Computational Time
- Instances with RL1

![Graph of Computational Time](image)
Computational Results

Nodes 22 RL 1.0

- Best Objective Value
- Worst Objective Value
- Objective Values
Computational Results

Nodes 22 RL 2.5

- MSMC\&WG
- MS NNG
- MC\&WGP
- NNGP
- MSTSPNNGS
- TSP NNGSP
- TSPLKSP

- Best Objective Value
- Worst Objective Value
- Objective Values
Computational Results

[Graph showing computational results for different algorithms with various bars and lines representing best, worst, and objective values.]
The main contributions of this works are:

▶ Introduction and mathematical formulation of a new Risk index;
▶ Introduction of a new unexplored variant of the vehicle routing problem named RCTVRP;
▶ Mathematical formulation of the RCTVRP;
▶ Exact approach to solve small instances of the problem;
▶ 7 metaheuristic approaches to solve the RCTVRP in a reasonable time.
Future research

- Completing the statistical analysis of the results obtained
- Handling the risk as an additional objective to be minimized
- Finding some real applications of the problem involving CIT companies
- Several extensions of the problem can be proposed for future research taking in consideration some real-life constraints as:
  - route length restrictions;
  - time windows;
  - precedence relations.
Thank you!
Questions?

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\[
\begin{align*}
\min \quad & \sum_{r=0}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} t_{ij} x_{ij}^r \\
\text{s.t.} \\
& \sum_{j=0}^{n} x_{0j}^r = \sum_{i=0}^{n} x_{i0}^r \quad r = 0, \ldots, m \\
& \sum_{j=0}^{n} x_{0j}^0 = 1 \\
& \sum_{i=0}^{n} x_{i0}^r \geq \sum_{j=0}^{n} x_{0j}^{r+1} \quad r = 0, \ldots, m - 1 \\
& n - \left( 1 + \sum_{r=0}^{m} \sum_{i=0}^{n} \sum_{j=0}^{n} x_{ij}^r \right) \leq n \cdot \sum_{h=1}^{n} x_{0h}^{r+1} \quad r = 0, \ldots, m - 1 \\
& \sum_{h=0}^{n} x_{hj}^r - \sum_{k=0}^{n} x_{jk}^r = 0 \quad j = 0, \ldots, n, \quad r = 0, \ldots, m
\end{align*}
\]
Mathematical Formulation 2/3

\[ P_0^r = 0 \quad r = 0, \ldots, m \quad (6a) \]

\[ P_i^r + p_j - \left(1 - x_{ij}^r\right) \cdot HP \leq P_j^r \quad i = 0, \ldots, n \quad j = 1, \ldots, n \quad r = 0, \ldots, m \quad (6b) \]

\[ 0 \leq P_i^r \leq \sum_{j=0}^{n} x_{ij}^r \cdot \sum_{i=0}^{n} p_i \quad i = 0, \ldots, n \quad (6c) \]

\[ R_j^r \geq P_0^r \cdot t_{0j} - \left(1 - x_{0j}^r\right) \cdot HC \quad j = 1, \ldots, n \quad r = 0, \ldots, m \quad (7a) \]

\[ R_j^r \geq R_i^r + P_i^r \cdot t_{ij} - \left(1 - x_{ij}^r\right) \cdot HC \quad j = 1, \ldots, n \quad r = 0, \ldots, m \quad (7b) \]

\[ 0 \leq R_i^r \leq T \cdot \sum_{j=0}^{n} x_{ij}^r \quad i = 0, \ldots, n \quad r = 0, \ldots, m \quad (7c) \]
\[ u_i^r - u_j^r + n \cdot x_{ij}^r \leq n - \sum_{l=1}^{n} x_{0l}^r \]

\[ u_0^r = 0 \]

\[ 0 \leq u_i^r \leq (n - 1) \cdot \sum_{j=0}^{n} x_{ij}^r \]

\[ u_{ij}^r \in \{0,1\} \]

\[ x_{ij}^r \in \{0,1\} \]

\[ i = 0, \ldots, n \]
\[ j = 1, \ldots, n \]
\[ i \neq j \]
\[ r = 0, \ldots, m \]

(8a)

\[ u_0^r = 0 \]

\[ r = 0, \ldots, m \]

(8b)

\[ 0 \leq u_i^r \leq (n - 1) \cdot \sum_{j=0}^{n} x_{ij}^r \]

\[ i = 1, \ldots, n \]
\[ r = 0, \ldots, m \]

(8c)

\[ u_{ij}^r \in \{0,1\} \]

\[ i = 0, \ldots, n \]
\[ j = 1, \ldots, n \]
\[ r = 0, \ldots, m \]

(8d)

\[ x_{ij}^r \in \{0,1\} \]

\[ i = 0, \ldots, n \]
\[ j = 1, \ldots, n \]
\[ r = 0, \ldots, m \]

(9)