Efficient metaheuristics to solve the intermodal terminal location problem

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1. Introduction

After the establishment of the Kyoto protocol in 1997, Europe dedicated itself to a significant reduction in the emission of greenhouse gases. The aim was to reduce the general amount of greenhouse gases in the atmosphere by 8% compared to 1990 in the period 2008–2012. Although the EU27 had already achieved a general reduction of 7.3% by 2006, the reduction in CO₂-emissions was a lot less promising. This lack of a significant decrease in CO₂ can be attributed almost entirely to the transportation sector which accounts for 30% of all CO₂-emissions. Moreover, road transportation is by far the most polluting transport mode, responsible for 71% of all transport CO₂-emissions [30]. These numbers clearly show the need to reduce the environmental impact of transportation activities in general, and to reduce CO₂-emissions stemming from road transportation in particular.

Multimodal transportation is one of the most promising approaches to achieve these goals [7]. It is defined as the transportation of goods by at least two different modes of transport [31]. This paper focuses on a particular type of multimodal transportation called intermodal transportation. This term is used for the multimodal transport of goods in one and the same intermodal transport unit by successive modes of transport, without handling of the goods themselves when changing modes. An excellent example of an intermodal transportation network is the worldwide transportation network of containers. Because containers are highly standardized, they can be shipped from and to virtually any place in the world through an enormous network of transshipment terminals (ports, railway stations) and by many different transport modes (ships, barges, trains, trucks).

Since different transport modes have different environmental profiles, i.e., their burden on the environment per tonne-kilometer differs significantly, combining transport modes offers new opportunities for reducing the carbon footprint of the transportation sector. Nevertheless, intermodal transportation is still a long way from becoming an economically viable alternative for road transportation [18]. In industry, modal choices are mainly based on economical criteria such as cost, flexibility and service level. For small to medium distances, no other transport mode (or combination of transport modes) can currently compete with road transportation on these criteria.

The low level of maturity of intermodal transportation in Europe can be explained to large extent by the poor intermodal infrastructure and connectivity of transport modes. Since the initial drayage move and terminal operations contribute in a significant way to transit time and costs of intermodal transport services, the location of the intermodal terminals plays an important role in enhancing the attractiveness of intermodal transportation [26]. An intermodal freight terminal is defined as a location equipped for transshipment of intermodal transport units (ITUs) between modes [31]. The policy decision where to locate new terminals is a complicated one, often involving many stakeholders (e.g. terminal operators, freight operators, local communities, investors and policy makers) with their own objectives [28] and supporting these decisions by adequate quantitative methods requires considerable simplification of reality.
This paper uses the model of Arnold et al. [2], in which the intermodal terminal location problem is formulated as a mixed-integer programming problem. In their model, a set of potential terminal locations is given, each with a given capacity and a fixed cost that is incurred when the terminal is opened. Additionally, a set of customers is given, representing origins and destinations of demand that is to be transported through the network from its origin to its destination. For each demand (i.e., each pair of an origin customer and a destination customer), the unimodal transportation cost is given, as well as the cost of routing this demand via any pair of terminals. In this model two decisions need to be taken simultaneously. The first decision is which of the set of potential terminals to open. The second is how to route each of the individual demands through the network. This can be done in a unimodal way, i.e., directly from the origin to the destination customer, or in an intermodal way, by transporting it from the origin customer to a terminal, then to a terminal closer to the destination customer and finally to the destination customer itself. The objective of the model developed by Arnold et al. [2] is to minimize the total cost, which is the sum of fixed costs for opening terminals and the cost of routing all demands through the network. A more detailed and formal description of this problem is given in Section 3.

The model used in this paper and the methods that have been developed to solve it, can be used to support high-level decision making on the layout of an intermodal terminal transportation network. This decision occurs both in a policy-making context and a business context. For example, if the Belgian government wants to determine which of a set of potential railway stations for container transport it should build, the methods could be used to quickly find a good solution under several different scenarios. Similarly, if a large trading company wants to determine which existing ports it should use to ship its containers from its production plants to its customers, it could similarly employ the methods developed in this paper. However, notwithstanding the fact that the decision where to locate intermodal transshipment terminals is a long-term decision for which, in principle, significant calculation time is available, the heuristics developed in this paper are able to compute near-optimal solutions in very short computing times. The reason for this design choice is that the model used in this paper—as well as other mathematical models of similar problems—is only able to capture part of the complex societal and economical reality of the real decision situation. As a result, such complex decisions are usually not taken in a one-shot way, but are the result of careful analysis in which several scenarios are evaluated. For example, a courier company that is planning to use a certain airport as a hub will be interested in the consequences on its operations of a government restriction on night time flights. A mathematical model and optimization method can therefore only aid a decision maker by providing him with insight into the optimal locations of terminals under varying circumstances. The best way to support such decisions is by integrating the method in an interactive decision tool, where the decision maker can manually adjust the parameters of the model and quickly re-optimize. The underlying method of such a what-if analysis tool should be able to respond quickly to such parameter adjustment.

Arnold et al. [2] propose to solve their model using a branch-and-bound procedure. However, since this terminal location problem is NP-hard (see Appendix A), such exact algorithms can only be used for small instances. In order to solve real-life instances of the intermodal terminal location problem, it is therefore necessary to use faster, heuristic solution methods. The objective of this paper is to provide decision makers with a new, fast but effective method to solve the terminal location problem as described by Arnold et al. [2]. More specifically, two different heuristics are developed, based on respectively the attribute based hill climber (ABHC) and the GRASP metaheuristic frameworks. Both heuristics are compared to each other in terms of solution quality, speed and ease of use.

The remainder of this paper is organized as follows. In Section 2 the literature on the intermodal terminal location problem is surveyed and the similarities and differences with related optimization problems are discussed. Section 3 presents the mathematical model of the terminal location problem, for which two heuristic solution approaches are developed in Section 4. These approaches are tested in Section 5. Section 6 concludes and provides pointers for future research.

2. Literature

Although research on intermodal transportation is still in an early phase [5], the strategic importance of intermodal transshipment terminals within intermodal networks is demonstrated by the amount of literature that has been accorded to this subject in recent years. A wide variety of topics related to intermodal terminals has been discussed, ranging from the main characteristics and the different types of intermodal terminals (e.g. [28]) over measuring terminal performance (e.g. [12]) to an analysis of the terminal market and a description of its most important stakeholders (e.g. [35]).

Both the complexity and importance of optimally locating intermodal terminals within the transport network has been acknowledged early on by many authors stemming from different research fields. Macharis and Bontekoning [21] provide an overview of the most prominent research efforts within the field of operations research. They distinguish three different approaches to determine the optimal location of transshipment terminals. Although some authors propose the use of simulation techniques (e.g. [24]) and multi-criteria analysis (e.g. [22]) to select the most appropriate location from a pool of potential sites, the dominant approach is the application of network models. Ishfaq and Sox [17] provide an overview of several models in the literature, starting with the model of Arnold et al. [2] (of which a different formulation was later given in Arnold et al. [3]). Other contributions include the ones by Groothedde et al. [15], Racunica and Wynter [27], Limbourg and Jourquin [20], Ishfaq and Sox [16]. For full descriptions, see Ishfaq and Sox [17].

Several commonly investigated optimization problems are closely related to the intermodal terminal location problem. The capacitated (fixed charge) facility location problem (CFLP), for which metaheuristics are frequently used as a solution method, shows some similarities. The CFLP considers the problem of selecting a subset of facilities from an existing set of potential locations that have to supply a set of customers at a minimum cost. Each customer has an associated demand to be met and each terminal has a finite amount of supply available [33]. There are quite a number of differences however with the intermodal terminal location problem. Most importantly, demand cannot be transported between facilities. Different metaheuristics have been applied successfully to the CFLP, such as simulated annealing (e.g. [6]), genetic algorithms (e.g. [19]), tabu search (e.g. [13]) and ant colony optimization (e.g. [32]).

The multicommodity capacitated (fixed charge) network design problem (MCNDP) [9,14], in which a set of demands have to be routed through a network, shows some similarities to the intermodal terminal location problem, but differs in that capacities are defined on arcs and not on nodes and that fixed costs have to be paid if an arc is used (instead of a node). This leads to quite different models and algorithms.
The intermodal terminal location problem discussed in this paper is related to the family of hub location problems (see [1] for a recent survey of hub location problems and algorithms to solve them). Many different hub location problems have been defined, but the multiple-allocation capacitated hub location problem with fixed costs comes closest to the problem discussed in this paper.

The main difference is in the way demand is routed through the network: hub location problems do not allow transportation between customers directly (i.e., they do not allow unimodal transportation), but do allow transport via only one hub, something that is not possible in the intermodal terminal location problem. The first LP formulation for the multiple allocation capacitated hub location problem (without fixed costs) is due to Campbell [8], but papers on capacitated hub location problems have been few and far between. An efficient algorithm for a capacitated hub location problem can be found in Ebery [11].

Boland [4] develops preprocessing procedures and tightening constraints. A different formulation and an algorithm can be found in Marin [23], who also confirm the scarceness of papers on this topic. Intermodal terminal location is a complicated decision with many stakeholders. Some authors attempt to capture the complex reality by presenting extensions of existing optimization models that include, e.g. non-linear cost functions [27]. Others have used agent-based techniques in which different stakeholders are represented by different agents (see [29], for an overview).

As mentioned, this paper takes the position that locating intermodal terminals is such a complicated decision that no model – however complex – is going to capture more than a fraction of the real problem. Any model that is going to be used in practice should therefore be simple and understandable and any method to solve it should be able to quickly locate a reasonable solution, so that the method can be embedded in a decision support system. For these reasons, the intermodal terminal location model presented by Arnold et al. [2] is used. To the best of our knowledge, this paper presents the first metaheuristic for this problem.

3. Model formulation

In Arnold et al. [2], the authors propose a model to determine the optimal rail/road network in Belgium by means of integer linear programming. The optimal network is defined as the network configuration that minimizes the total cost, i.e., the sum of uni- and intermodal transportation costs and fixed terminal location costs.

The model described in Arnold et al. [2], which is also used in this paper, can be described as follows. Let $I$ be the set of all origins/destinations and $K$ the set of all potential terminal locations in the network. Each origin/destination-pair ($ij$) has associated with it a positive and fixed amount $q_{ij}$ of goods that should be transported (the demand, with $q_{ij} = 0$), a decision variable $w_{ij}$ and a set of decision variables $x_{ij}^{km}$. The variable $w_{ij}$ represents the fraction of the demand $q_{ij}$ transported unimodally whereas the set of variables $x_{ij}^{km}$ relate to the fraction of the demand $q_{ij}$ shipped intermodally using terminals $k, m \in K$. $c_{ij}$ is the unit cost of transporting demand between $i$ and $j$ through terminals $k$ and $m$ and $c_{ij}$ is the unit cost of transporting demand directly from $i$ to $j$ without any intermediate intermodal operations. In turn, each potential terminal location $k \in K$ has associated with it a positive and fixed capacity $C_k$, a fixed cost $F_k$ and a decision variable $y_k$ which is equal to 1 when terminal $k$ is open and equal to 0 otherwise:

$$\text{min} \quad \sum_{i \in I} \sum_{j \in I} c_{ij} x_{ij}^{km} + \sum_{i \in I} c_{ij} w_{ij} + \sum_{k \in K} F_k y_k$$  \hspace{1cm} (1)

s.t. $x_{ij}^{km} \leq q_{ij} y_k \quad \forall k, m \in K, \quad \forall i \in I$  \hspace{1cm} (2)

$x_{ij}^{km} \leq q_{ij} y_m \quad \forall k, m \in K, \quad \forall i \in I$  \hspace{1cm} (3)

$\sum_{k,m \in K} x_{ij}^{km} + w_{ij} = q_{ij} \quad \forall i \in I$  \hspace{1cm} (4)

$\sum_{i \in I} \sum_{j \in I} x_{ij}^{km} + \sum_{i \in I} \sum_{j \in I} x_{ij}^{km} \leq C_k \quad \forall k \in K$  \hspace{1cm} (5)

$w_{ij} \geq 0, x_{ij}^{km} \geq 0, x_{ij}^{km} = 0 \quad \forall i \in I, \quad \forall k, m \in K$  \hspace{1cm} (6)

$y_{k} \in \{0, 1\} \quad \forall k \in K$  \hspace{1cm} (7)

The objective function (1) minimizes the total transportation cost associated with all transportation flows within the network. It is composed of three parts. The first part represents the cost of all transportation flows that require a change in transportation mode whereas the second part refers to the unimodal transportation flows. The third part equals the total fixed cost associated with all opened terminals in the network. Constraints (2) and (3) ensure that no goods can be transported using a certain terminal, unless this terminal is open. Constraints (4) stipulate that, for each origin/destination-pair, the sum of all goods transported unimodally and intermodally should equal the demand associated with this origin/destination-pair. Constraints (5) bring into account the limited capacity of the intermodal terminals. Constraints (6) ensure that only non-negative amounts are transported and that no demand is transported using only one terminal. Finally, constraints (7) enforce that a terminal should be fully used or not at all.

![Fig. 1. Example of the intermodal terminal location problem.](image-url)
The model can be used for policy-making purposes, in which both origins and destinations should be interpreted as zones of economic activity that ship goods through the network to each other. Each zone can be both a supply and a demand point at the same time.

A graphical representation for a simple problem with 3 customers, 3 origin-destination flows, and 3 terminals is shown in Fig. 1. All terminals are assumed to have a capacity of 100 units (indicated between brackets). The left part of the figure shows the (unimodal) transport streams if no terminals are open. The right part visualizes how these streams are rerouted when terminals $t_1$ and $t_2$ are open and $t_3$ is closed. The flow from $c_1$ to $c_2$ is still shipped by unimodal transport, whereas the flow between $c_1$ and $c_3$ is shipped entirely by intermodal transport. The flow between $c_2$ and $c_3$ is shipped only partially by intermodal transport, due to the limited capacity of the terminals.

In Appendix A a mathematical proof of the NP-hardness of this problem is provided.

4. Efficient metaheuristics for the intermodal terminal location problem

In this section, two different metaheuristics are proposed to solve the intermodal terminal location problem described in the previous section. Both heuristics consist of two phases: a construction phase (see Section 4.2) and an improvement phase (see Section 4.3). In the construction phase, initial solutions are built by a constructive metaheuristic and stored in an archive. The improvement phase – as the name suggests – improves the solutions in the archive. In both phases, the heuristics make use of a heuristic evaluation procedure to quickly evaluate solutions (see Section 4.1). The two metaheuristics differ in the way initial solutions are constructed and in the way the archive is maintained. The first metaheuristic uses a GRASP (greedy randomized adaptive search procedure) construction procedure, the second one an ABHC (attribute based hill climber). The local search heuristics used in the improvement phase are identical for both metaheuristics. Fig. 2 shows the high-level structure of the metaheuristics.

It is important to note that solutions in the archive should not only be of high quality, they should also be diverse to ensure that they are not all in the same basin of attraction (i.e., the improvement heuristics should not end up in the same local optimum when improving the solutions in the archive). The mechanisms for ensuring diversity, which is measured using the Hamming distance, differ depending on the constructive metaheuristic (GRASP or ABHC) used.

4.1. Problem decomposition and heuristic evaluation procedure

In the approach proposed in this paper, the terminal location problem is decomposed in a master and a subproblem. A solution of the master problem corresponds to a status open or closed for each potential terminal location, whereas the subproblem determines the way each demand is routed through the network, given the open terminals. Given a solution to the master problem, i.e., a set $\mathcal{K} \subseteq K$ of open terminals, the subproblem is the following LP:

$$
\min \sum_{ij} \sum_{km} c_{ij}^{km} x_{ij}^{km} + \sum_{ij} c_{ij} w_{ij} + F_X
$$

subject to:

$$
\sum_{k,m} x_{ij}^{km} + w_{ij} = q_{ij} \quad \forall i,j \in I
$$

$$
\sum_{ij} \sum_{km} x_{ij}^{km} + \sum_{ij} \sum_{km} x_{km}^{jk} \leq C_k \quad \forall k \in \mathcal{K}
$$

$$
w_{ij} x_{ij}^{km} \geq 0 \quad \forall i,j \in I, k \in \mathcal{K}, m \in \mathcal{K}
$$

The objective function value of an optimal solution of the subproblem in Eqs. (8)-(11) is equal to the total cost of the corresponding solution to the original problem. Unfortunately, for large instances, the number of decision variables and constraints in this LP becomes very large and it quickly becomes impractical or even impossible to solve the problem to optimality. Given the fact that the heuristics need to evaluate many solutions, both in the construction and the improvement phase, a fast heuristic evaluation procedure is developed to quickly calculate a heuristic objective function value. This value is an approximation of the exact objective function value of a master problem solution (i.e., the objective function value of the optimal solution of the subproblem LP).

The heuristic evaluation procedure works as follows. For each pair of customers, one needs to decide how the corresponding demand will be shipped, i.e., unimodally, intermodally, or using a combination of both. In order to minimize the total cost and taking into account the terminal capacity constraints, it is important to allocate the capacity of each terminal to those shipments that benefit the most from using it.

In order to achieve this, the heuristic evaluation procedure creates for each demand $q_{ij}$, i.e., for each pair of customers $(i,j)$ a list $P_{ij}$ of all routes $M_{ij}^{km}$ (connecting customers from $i$ to $j$ through terminals $k$ and $m$) for which the cost $c_{ij}^{km}$ of shipping the associated demand via this trajectory is smaller than the unimodal cost $c_{ij}$. All lists $P_{ij}$ are sorted in order of increasing cost, and the unimodal route with cost $c_{ij}$ is added in final position. All lists $P_{ij}$ therefore contain all paths to transport demand $q_{ij}$ through the network, in order of increasing cost. If the cheapest trajectory means moving the demand unimodally (i.e., $c_{ij} \leq c_{ij}^{km}$ $\forall k,m$), then $P_{ij}$ contains only one path. If this is not the case, the difference in cost between the best and the second best trajectory is calculated. This value is called the regret ($r_{ij}$) and is further used to determine the sequence in which demands should be assigned to terminals. Since demands with a large regret imply a significant increase in total cost if not assigned to their cheapest trajectory, they should therefore be assigned with large priority. A so-called regret list $R$ is created that contains all demands $q_{ij}$ sorted in decreasing order of regret. Starting with the first demand on the regret list and continuing down the list, each demand is then assigned in the

![Fig. 2. Structure of the metaheuristics.](image-url)
best possible way (determined by its list \( P_b \)) that is still available. If, for any origin/destination-pair, there is not enough terminal capacity available to assign all demand to its cheapest trajectory, the surplus demand is assigned to the second cheapest trajectory and so on until the unimodal route becomes the most profitable. A pseudo-code version of the heuristic evaluation procedure can be found in Algorithm 1.

The calculations of the lists \( P_b \) and \( R \) are performed before the actual optimization process in order not to slow down the optimization process later on. To calculate the regrets, since at this point it is not yet known which terminals will be open and which will be closed, a configuration in which all terminals are open is assumed.

Algorithm 1 (Heuristic evaluation procedure).

**Input:** \( \text{y} = \{y_k\} \) (status of each terminal), \( R \) (list of origin-destination pairs, sorted in decreasing order of regret), \( P_b = \{P_b\} \) (list of all possible paths for all possible terminal pairs, sorted in increasing order of cost)

**Output:** Heuristic objective function value \( \hat{f}(\text{y}) \)

create list \( C \) with capacity \( C_k \) of each terminal; \( \hat{f}(\text{y}) = \sum_k (F_k + y_k) \);

foreach \((i,j) \in R\) do

allocated \( \rightarrow \) false;
\( p \leftarrow 0;\)
\( q_y \leftarrow q_y;\)

while allocated \( = \) false and \( p < |P_b|\) do

\( t1 \leftarrow \) terminal with lowest \( C_k \) in route \( p;\)
\( t2 \leftarrow \) terminal with highest \( C_k \) in route \( p;\)

if \( t1 \) and \( t2 \) are open then

if \( C_{t1} \) and \( C_{t2} \geq q_y \) then

allocated \( \rightarrow \) true;
\( C_{t1} \leftarrow C_{t1} - q_y;\)
\( C_{t2} \leftarrow C_{t2} - q_y;\)
\( \hat{f}(\text{y}) \leftarrow \hat{f}(\text{y}) + C_k q_y;\)

else

\( \hat{f}(\text{y}) \leftarrow \hat{f}(\text{y}) + C_{t1} x \times q_y;\)
\( q_y \leftarrow q_y - C_{t1} + 1;\)
\( C_{t2} \leftarrow C_{t2} - C_{t1};\)
\( C_{t1} = 0;\)
\( p \leftarrow p + 1;\)

if allocated \( = \) false then

\( \hat{f}(\text{y}) \leftarrow \hat{f}(\text{y}) + c_{ij} q_y;\)

**4.2. Construction phase: GRASP or ABHC**

**4.2.1. GRASP**

GRASP, or greedy randomized adaptive search procedure, is a multi-start metaheuristic the objective of which is to build a feasible solution by starting from an empty solution and adding one element at a time. Typical of the GRASP metaheuristic is that it balances greedyness (selecting the best element at each iteration) and randomness (selecting a random element at each iteration) and is able to combine the advantages of both approaches. The GRASP heuristic for the terminal location problem starts from a solution in which all terminals are closed and opens one terminal at a time. A pseudo-code description of the GRASP algorithm is given in Algorithm 2.

Algorithm 2 (GRASP).

**Output:** Archive of diverse, high-quality solutions

create and sort list \( L \) with all terminals \( k \) according to \( r_k = \frac{F_k}{C_k} \) in ascending order;

while time available do

\( s_{\text{best}} \leftarrow \) solution \( s \) with all terminals closed;

foreach terminal \( k \in K \) do

create Restricted Candidate List \( T \) of terminals for which \( r_k \in [r_{\text{min}}, r_{\text{min}} + 2 \times (r_{\text{max}} - r_{\text{min}})] \) is true;

randomly select terminal \( t \in T;\)

remove \( t \) from \( L;\)

open \( t \) in \( s;\)

if \( f(s) < f(s_{\text{best}}) \) then

\( s_{\text{best}} \leftarrow s;\)

if size of archive < maximum archive size then

add \( s \) to archive;

else

if \( d(s) > \Delta \) then

if \( f(s) < f(s_{\text{closest}}) \) then

\( s \) replaces \( s_{\text{closest}} \) in archive;

\( s_{\text{best}} \) replaces \( s_{\text{closest}} \);

The order in which the terminals are opened is determined on the basis of the fixed cost/capacity-ratio of each terminal. For each terminal \( k \in K \), this ratio is calculated as \( r_k = F_k / C_k \). At each iteration, the GRASP algorithm determines a restricted candidate list (RCL) that consists of the best terminals, i.e., those terminals that have the lowest fixed cost/capacity-ratio. In this procedure, the restricted candidate list is composed of all terminals that satisfy the following constraint: \( r_k \in [r_{\text{min}}, r_{\text{min}} + 2 \times (r_{\text{max}} - r_{\text{min}})] \), with threshold parameter \( \Delta \in [0, 1] \). The terminal to be added is then randomly selected from the restricted candidate list and the objective value of the newly constructed solution is calculated using the heuristic evaluation procedure described earlier in this paper. Then, the restricted candidate list is updated to make sure that the element under consideration is no longer available for selection and the above process is repeated.

To preserve the diversity of the archive, a solution is only accepted into the archive if it fulfills two criteria. First, the constructed solution should differ sufficiently from all solutions currently in the archive. To determine the degree of difference between two solutions, the Hamming distance \( d \) is used, i.e., the number of terminals that are not in the same state (open or closed) in both solutions:

\[
d(s_1, s_2) = \sum_{k=1}^{K} (y_{s1}^k - y_{s2}^k)^2
\]

The GRASP procedure uses fixed threshold parameter \( \Delta \) and only allows a solution into the archive if its distance to each of the other solutions in the archive (denoted \( d(s) \)) is at least equal to \( \Delta \). The second criterion for addition is that the new solution should outperform its closest neighbor in the archive, \( f(s) < f(s_{\text{closest}}) \). A solution \( s \) that is added to the archive replaces \( s_{\text{closest}} \), its closest solution in terms of the Hamming distance.

The criteria for addition are overruled in one situation: when the new solution outperforms the best solution found so far during the construction phase. This solution, denoted as \( s_{\text{best}} \), is kept in memory at all times and added to the archive at the end of the GRASP-procedure, again replacing its closest archive-solution.
The GRASP algorithm has four parameters that need to be set: the threshold parameter \( x \) that determines the balance between randomness and greediness in the construction phase, the minimal distance from the other solutions in the archive \( D \), the maximum number of iterations, and the size of the archive. First of all, the aforementioned \( x \)-value needs to be set to determine the length and composition of the RCL.

The setting of this parameter \( x \) determines both the robustness and the quality of the constructed solutions. In order to determine the best possible \( x \)-value, a pilot study is performed in which the value of this parameter is systematically increased from \( x = 0 \) (completely greedy construction) to \( x = 1 \) (completely random construction) and investigated each of these cases on solution quality (objective value) and solution diversity (average solution distance). Based on the result of this study, \( x \) was fixed to 0.4 as for this value the best trade-off between both objectives was found. However, solution quality and diversity do not vary much with changing \( x \)-values, indicating that the method is rather robust.

Second, it is also necessary to specify the degree \( \Delta \) to which a solution should differ from all other solutions in the archive in order to be accepted in the archive. This decision is of great importance since it determines to large extent the diversity of the archive and has a large impact on the performance of the local search algorithm, especially its ability to locate different local optima. If the minimal distance is set to a very large value, it becomes very likely that the GRASP-procedure will be unable to find enough solutions that perform better than the unimodal optimum. If the minimal distance is set to a very large value, it becomes very likely that the GRASP-procedure will be unable to find enough solutions that perform better than the unimodal optimum. If the minimal distance is set to a very large value, it becomes very likely that the GRASP-procedure will be unable to find enough solutions that perform better than the unimodal optimum.

For each terminal location \( k \), the heuristic therefore maintains two solutions: the best solution found so far in which terminal \( k \) is open \((S_k)\) and the best solution found so far in which \( k \) is closed \((S^c_k)\). Each time a new solution is generated by adding a terminal, the following evaluation is made for all solution attributes: if terminal \( k \) is open \((S_k)\) and the best solution found so far with terminal \( k \) open, \( S_k \) is replaced by the new solution. If terminal \( k \) is closed, the comparison should be made with \( S^c_k \), i.e., the best solution found so far with terminal \( k \) closed. If the new solution is worse than both \( S_k \) and \( S^c_k \) for all values of \( k \), the terminal is closed again.

When all terminals have been either added or not, the ten best, non-duplicate solutions found during the search process are stored in an archive to be further improved later on using local search. In contrast to the GRASP-algorithm, the size of the archive is the only parameter to be set in the ABHC-procedure. Moreover, it is unnecessary to set up a complex archiving mechanism for the

### 4.2.2. Attribute based hill climber

The attribute based hill climber (ABHC) was recently proposed by Whitley and Smith [34] as a variant of the general tabu search algorithm. This metaheuristic is based on the aspiration concept and is shown to be competitive with the best-known approaches on a wide range of problems [34,10]. One of the most important features of ABHC is the fact that it is completely parameter-free.

The ABHC algorithm refines the aspiration criterion commonly found in tabu search algorithms and accepts a solution as the new current solution if it is the best solution found so far for at least one of its solution attributes. The ABHC heuristic (Algorithm 3) uses the same move type as the GRASP heuristic: in each iteration, the terminal with the lowest fixed cost-capacity ratio \( r_k \) and which is not yet open in the current solution, is added. The solution thus found is accepted as the new current solution if it is the best solution found so far for one of its solution attributes, defined as the status of a terminal.

**Algorithm 3 (Attribute based hill climber).**

**Output:** Archive of diverse, high-quality solutions

create and sort list \( L \) with all terminals \( k \) according to \( r_k = \frac{F_k}{c_k} \) in ascending order;

while improvement do

improvement ← false;

\( l \leftarrow 0; \)

while \( l < \text{size of } L \) do

select terminal \( l \in L \);

if terminal \( l \) is closed in solution \( s \) then

open \( l \) in solution \( s \);

foreach terminal \( k \in K \) do

if terminal \( k \) is open in solution \( s \) then

if \( f(s) < f(S_k) \) then

\( S_k \leftarrow s; \)

improvement ← true;

else

if \( f(s) < f(S^c_k) \) then

\( S^c_k \leftarrow s; \)

improvement ← true;

end if

end if

end for

\( l \leftarrow l + 1; \)

end if

insert the 10 best different solutions of \( s_i \) and \( S^c_i \) for all terminals \( i \in K \) into the archive

**Fig. 3.** Evaluation of the best value of \( \Delta \) (minimal distance to the archive) to achieve high-quality, diverse solutions.
ABHC heuristic since the archive solutions showed to be already sufficiently diverse in nature (see Fig. 5).

4.3. Improvement phase: ADD and REMOVE local search

After the construction phase, each solution in the archive is improved by an interchange heuristic that performs a local search (LS) on this solution. This local search heuristic consists of 2 steps, called ADD and REMOVE respectively. The first step (ADD) sorts the closed terminals in increasing order of \( r_k = F_k / C_k \) and attempts to open each terminal in that order. The terminal is opened if this decreases the cost of the solution and then removed from the list of possible terminals to be opened. The REMOVE step on the other hand sorts the open terminals in increasing order of \( r_k = F_k / C_k \) and attempts to close each open terminal in this order. The terminal is closed if this decreases the cost of the current solution. At the end of the REMOVE step, the improved solution is added to the archive.

At the end of the improvement phase, the best solution in the archive is returned.

5. Performance evaluation

5.1. Problem instance generation and exact solutions

Because instances for the intermodal terminal location problem are not available, a simple instance generator has been designed and implemented. To generate an instance, both the number of customers and the number of potential terminal locations in the intermodal network are first determined. For each customer \( i \), coordinates \( (x_i, y_i) \) are randomly generated in the Euclidean square between \( (0,0) \) and \( (0,0,10000) \). The amount of goods (the demand) \( q_{ij} \) to be shipped between each pair of customers \( (i,j), i \neq j \) is randomly generated in the interval \( [0,0,500] \). The \( x \)- and \( y \)-coordinates of the terminals are generated in the same manner as those of the customers. The fixed cost \( F_k \) and capacity \( C_k \) of each terminal \( k \) are randomly drawn from the intervals \( [0,10000] \) and \( [0,500] \) respectively.

The unimodal cost of a route \( c_{ij} \) is equal to the Euclidean distance between customers \( i \) and \( j \). To determine the cost of multimodal transport, the direct distances between customer \( i \) and the departing terminal are summed up, the distance between the two terminals and the distance between the receiving terminal and customer \( j \). To incorporate discounts for using multimodal transport, this sum is then divided by two.

In order to assess the quality of the methods proposed in this paper, 100 problem instances were generated in this way. An instance is generated for each possible combination of number of customers and terminals, in which both the number of customers and the number of terminal locations can vary from 10 to 100 in steps of 10. The instances can be downloaded from http://antor.ua.ac.be/intermodal.

All instances were solved by implementing the MIP formulation in Eqs. (1)–(7) in version 3.0.1 of Gurobi (http://www.gurobi.org). Gurobi was able to optimally solve 53 out of 100 instances in computing times ranging from 0.2 to 872 s. All experiments were performed on an Intel(R) Core(TM)2 Quad CPU Q6600 at 2.40 GHz with 3.2Gb RAM. Detailed results can be found on http://antor.ua.ac.be/intermodal. The first column contains the name of the instance, where RCLN indicates an instance with \( n \) customer and \( l \) potential terminal locations.

5.2. Performance evaluation of the heuristic evaluation procedure

Assessing the quality of the heuristic evaluation procedure developed for objective value calculation is done on the basis of the Kendall tau rank correlation coefficient \( (\tau) \) that measures the correspondence between two rankings and assesses its significance. This coefficient can take on any value between –1 and 1, where \( \tau = 1 \) corresponds to a situation in which the two rankings are identical and \( \tau = -1 \) to a situation in which one ranking is exactly the opposite of the other.

Kendall’s coefficient has been calculated for 40 of the 100 problem instances. For each of these instances, 1000 random solutions of the master problem are generated in the following way. First, the number of open terminals \( n_r \) is randomly chosen between 0 and \( n \). Next, \( n_r \) terminals are randomly chosen from the list of possible terminals and opened. The subproblem for these solutions is then solved exactly (using the formulation in Eqs. (8)–(11)) and heuristically (using the heuristic evaluation procedure). The exact objective functions were calculated using Gurobi 3.0.1. The remaining 60 problem instances were too large for Gurobi to be able to calculate exact objective function values for each number of terminals open, rendering them unusable for the purpose of this section. For each instance the 1000 solutions were ranked according to their exact objective function values and according to their heuristic objective function values and these rankings were compared using Kendall’s \( \tau \).

A correlation coefficient \( \tau \) close to 1 is a strong indication that the best solutions are also listed among the best solutions found by the heuristic evaluation procedure. In turn, this would imply that the heuristic evaluation procedure performs well at determining whether a given solution is better than another, an evaluation that has to be made several times during the optimization process. A sample of the results of these calculations can be found in Table 1. A scatter plot of the objective function values and ranks of 1000 solutions for a selected example is shown in Fig. 4.

The test results show that – overall – the heuristic evaluation procedure performs well. In some cases however, such as instance 50C10L, Table 1 shows a rather low correlation coefficient. A more detailed analysis of these cases unmasked that they all share the same properties: low terminal prices in combination with a very low ratio of the terminal capacity and the amount of goods to be shipped through the network. In such situations, it is less important which terminals are opened as (1) opening terminals is cheap and (2) only a fraction of the demand will be shipped by intermodal transport anyway (because of the low terminal capacity). Consequently, for these cases, the objective value of any solution will not deviate much from that of

<table>
<thead>
<tr>
<th>C</th>
<th>L</th>
<th>( \tau )</th>
<th>( \Delta_{\max}^* ) (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>0.89</td>
<td>22.24</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>0.88</td>
<td>20.87</td>
</tr>
<tr>
<td>10</td>
<td>50</td>
<td>0.89</td>
<td>20.28</td>
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<tr>
<td>20</td>
<td>10</td>
<td>0.84</td>
<td>5.15</td>
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<td>30</td>
<td>0.88</td>
<td>20.75</td>
</tr>
<tr>
<td>20</td>
<td>50</td>
<td>0.89</td>
<td>26.17</td>
</tr>
<tr>
<td>30</td>
<td>10</td>
<td>0.77</td>
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<td>0.86</td>
<td>15.52</td>
</tr>
<tr>
<td>30</td>
<td>50</td>
<td>0.93</td>
<td>23.73</td>
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<tr>
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<td>0.13</td>
<td>3.91</td>
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<tr>
<td>40</td>
<td>30</td>
<td>0.66</td>
<td>10.64</td>
</tr>
<tr>
<td>40</td>
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<td>0.92</td>
<td>18.62</td>
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<tr>
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<tr>
<td>50</td>
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<td>0.10</td>
<td>7.93</td>
</tr>
</tbody>
</table>

* Difference between the exact objective function values best and the worst solution of 1000 random solutions.
optimal solution, i.e., in these cases almost all solutions are good solutions. Indeed, Table 1 indicates that the absolute difference between the worst possible solution and the best is very small in these cases. Therefore, despite a low $\tau$, this evaluation method still allows the overall metaheuristic to make good decisions.

Finally, it should be noted that there can be significant deviations between the exact and the heuristic objective function value of a solution, even for problem instances where the value of $\tau$ is close to 1. The high value of $\tau$ means that the heuristic will in most cases correctly judge whether one solution of the master problem is better than another, although it may be preferable to use a slower heuristic to solve the subproblem (assign demands to terminals) for the final solution of the master problem.

5.3. Metaheuristics performance evaluation

In this section, the relative performance of GRASP and ABHC is compared, before and after local search. Detailed result tables can be found on http://antor.ua.ac.be/intermodal.

In a first phase, both solution construction methods are allowed the same computing time. Since the ABHC has a natural stopping criterion, the GRASP-procedure is allowed to run exactly as long as the ABHC method (GRASP$_a$). First, the archive of both methods is compared in terms of solution quality and diversity before local search. If the solutions in the archive are already of high quality at this point, it will take the local search algorithm less time to reach a local optimum. Additionally, the more diverse the solutions in the archive, the more local optima can be found by the local search improvement heuristic and the greater the probability of reaching the global optimum.

The solution quality is evaluated by comparing the objective function value of the best solution in the archive of both methods. To compare the solution diversity in the archive, the average of all average distances of each solution is compared to all other solutions in the archive. The results of these calculations can be found in Figs. 5 and 6. From these figures, it is clear that – at least before the local search phase – the ABHC-algorithm strongly outperforms the GRASP$_a$-algorithm on both parameters.

A similar comparison is made after local search, now in terms of solution quality and computing time. While the ABHC heuristic has a natural stopping criterion, the GRASP heuristic has not. To find the influence of limiting the computing time of GRASP$_a$ to that of ABHC, the GRASP$_b$ algorithm is allowed to run until the best solution has not been improved for 10 iterations, keeping the $\tau$ value unchanged. Figs. 7 and 8 show the comparison of both GRASP configurations with the ABHC heuristic, after local search. In Fig. 7 bars above the x-axis indicate that the GRASP$_a$+LS algorithm outperforms the ABHC+LS algorithm. This figure shows that the performance of the metaheuristic using GRASP$_a$+LS is on par with that using ABHC+LS. In this case, the solution quality gap between both
methods becomes almost non-existent, outliers have disappeared and the number of cases in which GRASP_α + LS performs better than ABHC + LS is more or less equal to the number of cases in which the reverse is true. The graph also shows no sufficient improvement of the solutions produced by GRASP_β to compensate for the longer calculation times.

It can be concluded from the previous analysis that there is no real difference in performance between the two metaheuristics developed in this paper. In a very short amount of time, they both provide the decision maker with high-quality solutions that do not deviate much in objective value from the exact solutions. The results of comparing the objective value of the best solutions generated by the heuristics and the optimal solutions are visualized in Fig. 9. In fact, the largest relative difference in total cost between the exact method and the metaheuristics using either ABHC, GRASP_α, or GRASP_β during the construction phase amounted to no more than 2.75%, 4.1% or 3.61% respectively. One argument in favor of using ABHC during the construction phase is that it is parameter-free. By avoiding the parametric discussion, this method is more transparent and hence more likely to be easily accepted by its users.

6. Conclusion and future research

The low level of maturity in intermodal transportation can be explained to a large extent by the poor intermodal infrastructure.
Especially the interconnectivity of the different transportation modes proves to be a large drawback in practice, resulting in high transit times and cost. Intermodal transshipment terminals play an essential role in enhancing the connectivity between modes and hence in improving the attractiveness of intermodal transportation in general.

In order to determine the optimal number and location of transshipment terminals in an intermodal network, two different metaheuristics have been developed that both consist of two phases: a construction heuristic to build solutions and a local search algorithm to improve solutions. The proposed methods only differ in terms of the applied construction heuristic: whereas the first method makes use of the GRASP metaheuristic, the second uses the relatively unknown ABHC (attribute based hill climber) instead. To allow the method to solve arbitrarily large instances in reasonable computing times, a heuristic evaluation procedure has been developed to calculate a good approximation of the objective function value in a short computing time.

The main purpose of this paper was to investigate which of the two methods performs best at solving real-life sized instances of the intermodal terminal location problem. These numerical results show however that both methods generate equally good solutions that approximate the optimal solution very closely. From a user perspective, one could argue that the ABHC heuristic should be preferred because it is parameter-free. Either way, tests clearly prove the usefulness and value of metaheuristics for solving real-life problems that are too complex to be solved to optimality.

This paper concludes with a couple of recommendations for further refinement of the heuristics and future research. Although increasing the number of iterations of the GRASP-algorithm had no significant impact on the solution quality, it remains interesting to investigate what the effect of further optimization of the GRASP parameter setting would be, for example by using techniques such as experimental design. The surplus time and effort that an in-depth parametric analysis demands, might not be justifiable for the time being, i.e., the tool serves as a trigger for discussion, but this attitude might change in the future when intermodalism would have achieved a higher level of maturity and the tool would indeed be used for determining the actual location of new intermodal terminals. Additionally, given the number of stakeholders involved in this problem and the impact on society of transportation in general, it is likely that the objective function, i.e., minimizing the total cost, is too simple to reflect reality and that the problem at hand should be transformed in a multi-objective one. In this way, the fixed cost for locating terminals could be split off from the variable transportation cost. An even more interesting alternative is to extend the objective function to incorporate other decision criteria such as environmental impact.

**Appendix A. Complexity of the intermodal terminal location problem**

**Theorem 1.** The intermodal terminal location problem in Eqs. (1)–(7) is NP-hard.

**Proof.** This result follows from the fact that the intermodal terminal location problem is an extension of the capacitated facility location problem, a known NP-hard problem [25]. This is shown by demonstrating that each instance of the CFLP can be converted to a corresponding instance of the intermodal terminal location problem in polynomial time.

Assume an instance of the CFLP with a set of customers $I_o$ and a set of potential facilities $K_o$. Each facility $k \in K_o$ has a fixed cost $F_k$ and a capacity $C_k$. Each customer $i \in I_o$ has a demand $d_i$ that has to be delivered from one or more of the facilities. The cost of transporting one unit of demand from facility $k$ to customer $i$ is equal to $c_{ik}$. If variables $x_{ik}$ are defined to be the demand transported from facility $k$ to customer $i$ and $y_k$ the binary variable that is equal to 1 if facility $k$ is used and zero otherwise, the mathematical formulation of this problem is the following:

$$\begin{align*}
\min & \sum_{i \in I_o} \sum_{k \in K_o} c_{ik}x_{ik} + \sum_{k \in K_o} F_k y_k \\
s.t. & x_{ik} \leq \tilde{q}_i y_k, \quad \forall i \in I_o, \quad \forall k \in K_o \\
& \sum_{k \in K_o} x_{ik} = \tilde{d}_i, \quad \forall i \in I_o \\
& \sum_{i \in I_o} x_{ik} \leq C_k, \quad \forall k \in K_o \\
& x_{ik} \geq 0, \quad \forall i \in I_o, \quad \forall k \in K_o \\
& y_k \in \{0, 1\}, \quad \forall k \in K_o
\end{align*}$$

This instance of the CFLP can be transformed into an instance of the intermodal terminal location problem. For each customer $i$ in the CFLP, two customers are created in the intermodal terminal location instance, an origin customer $i$ and a destination customer $\delta(i)$ ($\delta(i)$ is a function that returns the index of the destination customer corresponding to the origin customer in its argument). The demand to be transported from customer $i$ to customer $\delta(i)$ is set to equal the demand of customer $i$ in the CFLP instance, i.e., $q_{d(i)} \equiv \tilde{d}_i$. The demand from customer $\delta(i)$ to customer $i$ is also set to zero ($q_{o(i)} \equiv 0$), as well as all other demands between
customers not descendent from the same customer in the CFLP, i.e., \( q_{i|q_j} = 0, i \neq j \).

For each facility \( k \) in the CFLP, an origin and a destination terminal \( \delta(k) \) are created in the intermodal terminal location instance \( \delta(i) \) is a function that returns the destination terminal corresponding to the origin terminal in its argument) and set the capacity of these terminals both equal to the capacity of facility \( k \) in the CFLP, i.e., \( C_k = C_{\delta(k)} = C_k \). The fixed cost of locating terminals \( k \) and \( \delta(k) \) is set to half the fixed cost of locating facility \( k \) in the CFLP, i.e., \( F_k = F_{\delta(k)} = F_k/2 \). The cost of transporting a unit of demand from customer \( i \) to customer \( \delta(i) \) through terminals \( k \) and \( \delta(k) \) is set equal to the cost of delivering a unit of demand from facility \( k \) to customer \( i \) in the CFLP, i.e., \( c_{d(i)k} = c_{\delta(i)k} \). All other transportation costs, i.e., the cost of delivering demand between customers not descendent from the same customer in the CFLP, as well as the unimodal costs between any pair of customers \( c_{ij} \), are set to infinity.

From the above it follows that any optimal solution of the intermodal terminal location instance created in this way will have the following properties. First, transportation variables \( x_{d(i)k}^o \) will be zero if transportation is not from an origin to a destination customer corresponding to the same customer in the CFLP, using an origin and a destination terminal corresponding to the same facility. Also, all unimodal demand variables \( w_{ik} \) will be zero in the optimal solution. In other words, the only variables \( x_{d(i)k}^o \) that will be non-zero in the optimal solution are of the form \( x_{d(i)k}^o \). Since demand can only be shipped through terminals \( k \) and \( \delta(k) \) if both are open, it follows that in the optimal solution – terminals will only be opened in pairs. Opening an origin terminal without opening the corresponding destination terminal will only incur a cost without allowing any transportation. Therefore, in the optimal solution, \( y_k = F_{\delta(k)} \).

The mathematical formulation of the intermodal terminal location problem in Eqs. (1)–(7) for the instance generated using this special structure can thus be written as follows:

\[
\min \sum_{i} \sum_{k \in K} c_{d(i)k} \cdot x_{d(i)k} + \sum_{k \in K} F_k (y_k + y_{\delta(k)}) \tag{19}
\]

s.t.

\[
x_{d(i)k}^o \leq q_{i|q_j} y_k, \quad \forall k \in K_o \tag{20}
\]

\[
x_{\delta(i)k}^o \leq q_{\delta(i)j} y_k, \quad \forall k \in K_o \tag{21}
\]

\[
\sum_{k \in K_{\delta(i)}} x_{d(i)k}^o = q_{i|q_j}, \quad \forall i \in I_o \tag{22}
\]

\[
\sum_{k \in K_{\delta(i)}} x_{\delta(i)k}^o \leq C_k, \quad \forall k \in K_o \tag{23}
\]

\[
x_{d(i)k}^o \geq 0, \quad \forall i \in I_o, \forall k \in K_o \tag{24}
\]

\[
y_k, y_{\delta(k)} \in [0, 1], \quad \forall k \in K_o \tag{25}
\]

Since \( y_{\delta(k)} \equiv y_k \) in the optimal solution, Eq. (21) is redundant. After the following substitutions: \( x_{d(i)k} \rightarrow x_{d(i)k}^o, \quad y_{\delta(k)} \rightarrow y_k \), \( x_{\delta(i)k} \rightarrow x_{\delta(i)k}^o, \quad q_{i|q_j} \rightarrow q_{i|q_j}/2 \), the formulation of the CFLP instance that was converted to an intermodal terminal location problem instance is obtained.

This appendix shows that the CFLP is a special case of the intermodal terminal location problem. Therefore, since the CFLP is NP hard, so is the intermodal terminal location problem.

References


