University of Antwerp
Operations Research Group
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Understanding people's preferences:
an integrated algorithm for the optimal design of discrete choice experiments with partial profiles

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comex
combinatorial optimization:
metaheuristics & exact methods

Context (example)

Attributes	Dell Inspiron 15 i15	Lenovo IdeaPad Y510p
Screen size	15.6 inches	15.6 inches
Max. screen resol.	1366 x 768 pixels	1920 x 1080 pixels
Processor	1.9 GHz Intel Pentium 2127U ULV	2.4 GHz Intel Core i7-4700MQ
RAM memory	4 GB DDR3	16 GB DDR3
Hard drive	500 GB SATA	1 TB SATA

Objective: to determine the attributes that drive people's preferences

Discrete choice experiments (DCE)

Elements:

- Attributes
- Levels of each attribute
- Choice situations
- Options/choice situation

Assumption: utility theory

Objective: to determine the utility that consumers give to each attribute

DCE (example)

Attributes:

- Screen size
  - 13 inches
  - 15.6 inches
  - 17 inches
- Max. screen resolution
  - 1366 x 768 pixels
  - 1920 x 1080 pixels
- Processor
  - 2.0 GHz Intel Core i5
  - 2.6 GHz Intel Core i7
  - 2.6 GHz Intel Core i7
- RAM memory
  - 4 GB
  - 8 GB
  - 12 GB
- Hard drive
  - 500 GB
  - 750 GB
  - 1 TB

Choice situations: 30

Options/choice situation: 2

Choice situation (example)

Attributes	Option # 1	Option # 2
Screen size	13 inches	15.6 inches
Max. screen resol.	1366 x 768 pixels	1920 x 1080 pixels
Processors	2.0 GHz Intel Core i5	2.6 GHz Intel Core i7
RAM memory	4 GB	8 GB
Hard drive	1 TB	500 GB

Multinomial logit (MNL) model

Probability that a respondent chooses an option j in choice situation i:

$$p_{i,j} = \frac{e^{\mathbf{x}'_{\mathbf{i,j}}\boldsymbol{\beta}}}{\sum_{k=1}^{J} e^{\mathbf{x}'_{\mathbf{i,k}}\boldsymbol{\beta}}} \tag{1}$$

where:

- J is the number of options in each choice situation
- $\mathbf{x}'_{\mathbf{i,j}}$ describes option j in choice set i
- $\boldsymbol{\beta}$ contains the respondent's utilities for each attribute

Information matrix

defined as:

$$\mathbf{M}(\mathbf{X}, \beta) = \sum_{s=1}^{S} \mathbf{X}_\mathbf{s}'(\mathbf{P}_\mathbf{s} - \mathbf{p}_\mathbf{s}\mathbf{p}_\mathbf{s}')\mathbf{X}_\mathbf{s} \tag{2}$$

where:

- S is the number of choice options
- $\mathbf{X}_\mathbf{s}$ describes the options in choice situation s
- $\mathbf{p}_\mathbf{s} = [p_{s,1}, \ldots, p_{s,J}]'$
- $\mathbf{P}_\mathbf{s} = \text{diag}[p_{s,1}, \ldots, p_{s,J}]$

Cognitive behaviour

Assumption: humans make compensatory decisions

- This is a problem when there is:
  - inconsistent choice behaviour
  - lexicographic choice behaviour and dominant attributes
- Which occurs when:
  - there are too many attributes
  - the cognitive effort required is overwhelming

DCE with partial profiles

Attributes	Option # 1	Option # 2
Screen size	13 inches	15.6 inches
Max. screen resol.	1920 x 1080 pixels	1920 x 1080 pixels
Processors	2.0 GHz Intel Core i5	2.6 GHz Intel Core i7
RAM memory	4 GB	8 GB
Hard drive	1 TB	1 TB

Generation of optimal designs for DCE with partial profiles

Two decisions (for each choice situation):

- Set of constant attributes
- Levels of the varying attributes

$\mathcal{D}$-optimality criterion: maximize $\log |\mathbf{M}(\mathbf{X}, \beta)|$

- Utility neutral designs: $\boldsymbol{\beta} = \mathbf{0_K}$
- Locally optimal designs: specific $\boldsymbol{\beta}$ vector
- Bayesian optimal designs: prior distribution of $\boldsymbol{\beta}$ parameters (usually multivariate normal distribution)

Two-stage algorithm *(Kessels et al. (2012))*

First stage: determine the attributes to be held constant in each choice situation

Attributes	Option # 1	Option # 2
1		
2	x	x
3		
1	x	x
2		
3		
1		
2		
3	x	x
1	x	x
2		
3		

Two-stage algorithm *(Kessels et al. (2012))*

Second stage: optimize the levels of the varying attributes (coordinate-exchange algorithm)

Attributes	Option # 1	Option # 2
1	1	2
2	x	x
3	2	1
1	x	x
2	1	2
3	3	1
1	2	1
2	2	1
3	x	x
1	x	x
2	1	2
3	1	3

Integrated algorithm

Initial random design:

Attributes	Option # 1	Option # 2
1	2	1
2	x	x
3	3	2
1	1	1
2	1	2
3	x	x
1	1	2
2	2	2
3	x	x
1	1	2
2	x	x
3	1	3

Integrated algorithm

- **First component:** optimize the selection of attributes that are held constant
- **Second component:** optimize the levels of the varying attributes (coordinate-exchange algorithm)

Attributes	Option # 1	Option # 2
1	2	1
2	x	x
3	3	2

Integrated algorithm (exhaustive)

- Propagate the effects of the selection of constant attributes (in one choice situation)
- Recursive execution of the integrated algorithm

Attributes	Option # 1	Option # 2	
1	2	1	CS being optimized
2	×	×	
3	3	2	
1	1	1	
2	1	2	
3	×	×	
1	1	2	
2	2	2	
3	×	×	

Computational experiments

Benchmark UN designs:

Algorithm	$\mathcal{D}$-efficiency Min.	$\mathcal{D}$-efficiency Avg.	$\mathcal{D}$-efficiency Max.	# Optimal des.	Exec. time
Two-stage	88.45	97.51	100.00	2/50	-
Integrated	99.56	99.96	100.00	28/50	0.10
Integrated (e)	99.90	99.99	100.00	49/50	59.37

Computational experiments

UN designs under varying conditions:

	$\mathcal{D}$-efficiency			
Algorithm	Min.	Avg.	Max.	# Optimal des.
Two-stage	95.72	97.82	99.75	0/24
Integrated	98.11	99.67	100.00	1/24

Computational experiments

Locally optimal designs:

	$\mathcal{D}$-efficiency			
Algorithm	Min.	Avg.	Max.	# Optimal des.
Two-stage	72.43	94.86	99.83	0/16
Integrated	93.63	98.35	99.92	0/16

Integrated algorithm: the larger the part-worth values, the larger the number of times the attribute is held constant

Computational experiments

Bayesian optimal designs:

	$\mathcal{D}$-efficiency			
Algorithm	Min.	Avg.	Max.	# Optimal des.
Two-stage	93.16	97.30	99.42	0/13
Integrated	99.00	99.49	99.75	0/13

Integrated algorithm: the larger the variance, the larger the number of times the attribute is held constant (**counterintuitive)**

Conclusions

What we propose:

- A fast and flexible algorithm for the generation of DCE with partial profiles
- A slower but more effective exhaustive version

Important insights:

- Attributes with large part-worth values are held constant more often
- Attributes with large variances are held constant more often

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