The vehicle routing problem with backhauls (VRPB)

Two types of customers:
- The consumers (linehaul) that request goods from the depot
- The suppliers (backhaul) that send goods to the depot

Constraints:
- Every vehicle visits at least one linehaul customer
- In every route, neither the load of goods sent to the linehaul customers nor the load of goods received from the backhaul customers exceeds the vehicle capacity
- In every route, the linehaul customers are served before the backhaul customers

Iterated local search algorithm (ILS)

Initial Solution
Alternative procedures:
- Random: randomly insert the customers in the solution
- Greedy: iteratively insert the customer at the position that produces the smallest increase of the solution cost
- Initialization step: insert a randomly selected linehaul customer in every route

Oscillating local search (OLS)

- Consideration of solutions that violate the capacity constraint
- Neighborhood structure (each iteration):
  - Intra-route and inter-route customer relocation
  - Intra-route and inter-route customers exchange
  - Inter-route crossover
  - Intra-route 2-opt
- Cost function:
  \[ \text{cost}(S) = \text{distance}(S) + \alpha \sum_{1 \leq r \leq m} |\text{lh\_excess\_load}(r) + \text{bh\_excess\_load}(r)| \]
- Update rules for the penalization:
  - \( \alpha \) is initialized to a value \( \alpha_0 \) and multiplied by a factor \( \beta > 1 \) when the exploration process cannot find a better solution
  - When a locally optimal feasible solution is found, the OLS verifies whether the new solution is better than the best feasible solution found so far. If so, \( \alpha \) is set back to \( \alpha_0 \) and another complete cycle of the algorithm is executed

Perturbation
- Iteratively relocate customers in the solution
  - The customer and the new position are randomly selected

Statistical Analysis

Parameters studied:
- Procedure to generate the initial solution
- Perturbation size
- Initial penalty \( \alpha_0 \)
- Multiplicative factor used to increase the penalty \( \beta \)

Conditions
- 10 executions using each combination of parameter values
- Performance measures:
  - The cost of the best solution found (out of the 10 executions)
  - The average solution cost
  - The average execution time
- Estimation of a mixed-effects analysis of variance (ANOVA) model for each performance measure

Results
- Important parameters:
  - Size of the perturbation
  - Initial penalty \( \alpha_0 \) (OLS)
  - Best performance when:
    - Perturbation size is equal to 30%
    - The OLS is able to explore infeasible solutions with no restrictions (initial penalty \( \alpha_0 = 0 \))

ILS vs. state-of-the-art algorithms

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Table: Performance comparison of the algorithms and their scaled execution times.

Conclusions

- Features of the OLS heuristic:
  - Wide neighborhood structure (4 operators)
  - Exploration of solutions that violate the capacity constraint
- Important components of the ILS:
  - Perturbation size
  - Exploration of infeasible solutions (OLS)
- ILS vs. state-of-the-art algorithms:
  - Simpler and faster algorithm, better performance (2/3 measures)