An Iterated Local Search Algorithm for the Vehicle Routing Problem with Backhauls

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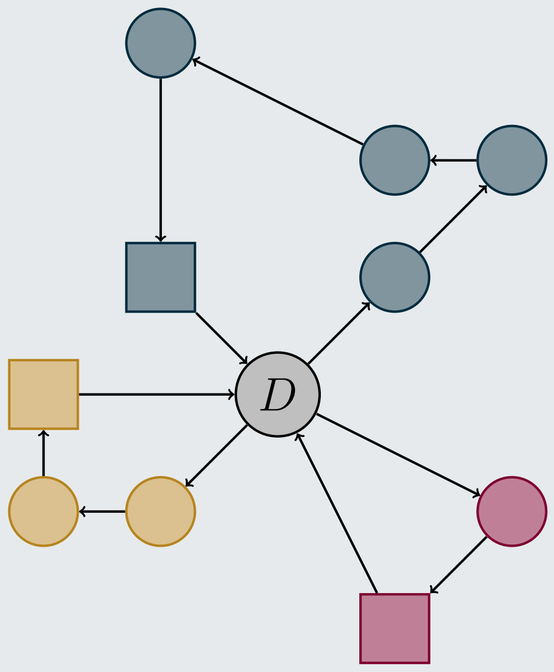
The vehicle routing problem with backhauls (VRPB)

Two types of customers:

- The consumers (**linehaul**) that request goods from the depot
- The suppliers (**backhaul**) that send goods to the depot

Constraints:

- Every vehicle visits at least one linehaul customer
- In every route, neither the load of goods sent to the linehaul customers nor the load of goods received from the backhaul customers exceeds the vehicle capacity
- In every route, the linehaul customers are served before the backhaul customers



Iterated local search algorithm (ILS)

Initial Solution

Alternative procedures:

- *Random:* randomly insert the customers in the solution
- *Greedy:* iteratively insert the customer at the position that produces the smallest increase of the solution cost
  - Initialization step: insert a randomly selected linehaul customer in every route

Oscillating local search (OLS)

- Consideration of solutions that violate the capacity constraint
- Neighborhood structure (each iteration):
  - Intra-route and inter-route customer relocation
  - Intra-route and inter-route customers exchange
  - Inter-route crossover
  - Intra-route 2-opt
- Cost function:

$$\text{cost}(S) = \text{distance}(S) + \alpha \sum_{1 \leq r \leq m} [\text{lh\_excess\_load}(r) + \text{bh\_excess\_load}(r)]$$

- Update rules for the penalization:
  - α is initialized to a value α_0 and multiplied by a factor $\beta > 1$ when the exploration process cannot find a better solution
  - When a locally optimal feasible solution is found, the OLS verifies whether the new solution is better than the best feasible solution found so far. If so, α is set back to α_0 and another complete cycle of the algorithm is executed

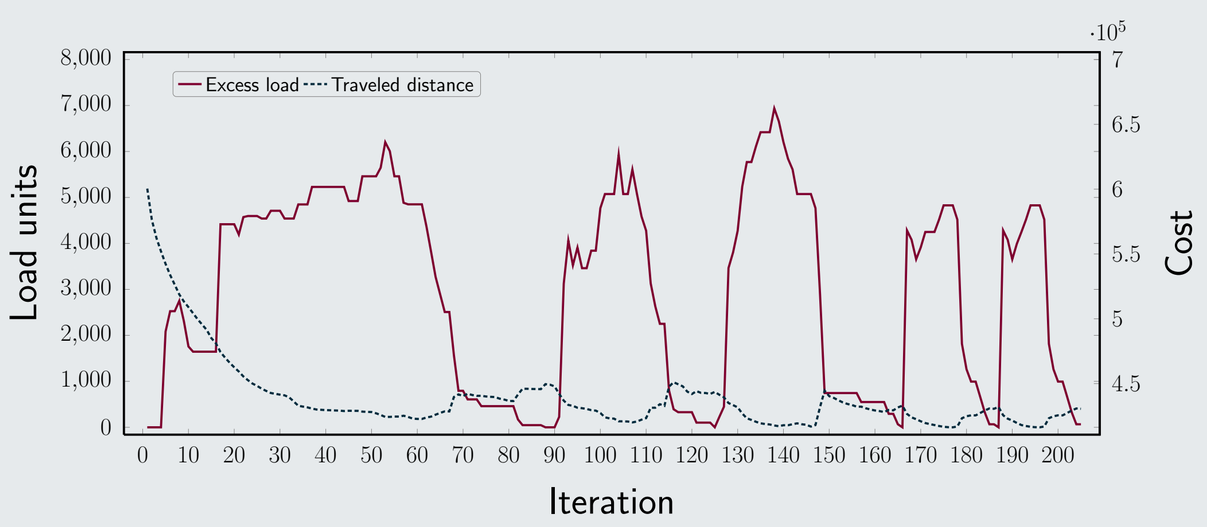


Figure : Search pattern of the OLS heuristic in terms of the excess load and the cost of the solutions explored at each iteration.

Perturbation

- Iteratively relocate customers in the solution
- The customer and the new position are randomly selected

Statistical Analysis

Parameters studied

- Procedure to generate the initial solution
- Perturbation size
- Initial penalty (α_0)
- Multiplicative factor used to increase the penalty (β)

Conditions

- 10 executions using each combination of parameter values
- Performance measures:
  - The cost of the best solution found (out of the 10 executions)
  - The average solution cost
  - The average execution time
- Estimation of a mixed-effects analysis of variance (ANOVA) model for each performance measure

Results

- Important parameters:
  - Size of the perturbation
  - Initial penalty α_0 (OLS)
- Best performance when:
  - Perturbation size is equal to 30%
  - The OLS is able to explore infeasible solutions with no restrictions (initial penalty $\alpha_0 = 0$)

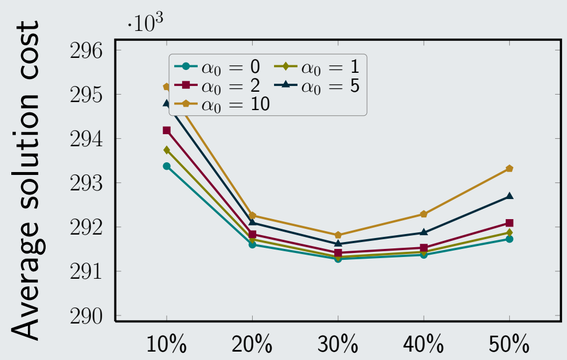


Figure : Influence of the perturbation size and the initial penalty α_0 on the average cost of the solutions found.

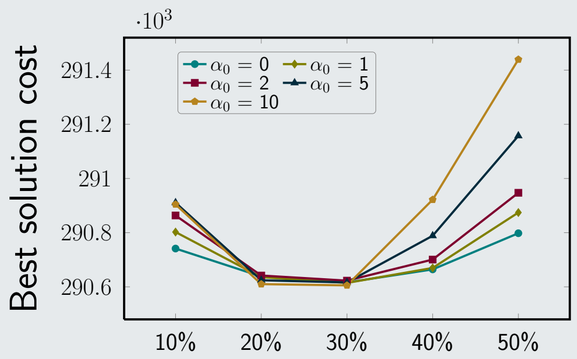


Figure : Influence of the perturbation size and the initial penalty α_0 on the average cost of the best solutions found.

ILS vs. state-of-the-art algorithms

Algorithm	Num. Best sol.	Avg. best sol. cost	Avg. sol. cost	Avg. time
Set JBG				
RTS-AMP	40/62	290981.80	-	11.01
BTS	39/62	291160.00	291305.70	36.67
LNS	50/62	291014.70	291823.34	14.48
MACS	46/62	290655.29	**290920.90**	37.35
ILS	**58/62**	**290593.84**	291332.41	14.31
Set TV				
RTS-AMP	21/33	706.40	-	4.33
BTS	25/33	702.20	702.50	13.36
LNS	26/33	701.18	704.50	8.54
MACS	27/33	701.48	**702.30**	14.17
ILS	**32/33**	**700.72**	704.42	3.83

Table : Performance comparison of the algorithms and their scaled execution times.

Conclusions

- Features of the OLS heuristic:
  - Wide neighborhood structure (4 operators)
  - Exploration of solutions that violate the capacity constraint
- Important components of the ILS:
  - Perturbation size
  - Exploration of infeasible solutions (OLS)
- ILS vs. state-of-the-art algorithms:
  - Simpler and faster algorithm, better performance (2/3 measures)