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REGULAR ARTICLE

A GRASP metaheuristic to improve accessibility after a disaster

Pablo Maya Duque · Kenneth Sörensen

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Abstract We consider the problem of allocating scarce resources to repair a rural road network after it has been damaged by a natural or man-made disaster. We propose a solution approach based on the GRASP and VNS metaheuristics that aims to maximize the accessibility of as many people as possible to the main cities or regional centers where the economic and social infrastructure is usually located. The efficiency of our approach is demonstrated by applying it to a set of small and medium size instances and to a large real-life motivated instance. Results point out the importance of OR techniques to support the disaster management decision process operations, particularly during the recovery phase.

Keywords Disaster relief · Greedy randomized adaptive search procedure (GRASP) · Variable Neighborhood Search (VNS)

1 Problem description

Natural disasters such as earthquakes, tsunamis, volcanic eruptions, hurricanes, typhoons, floods, and landslides have a huge impact on human life, as well as on the economy and the environment. In spite of the advances in forecasting and monitoring the natural hazards that cause those disasters, their consequences are often devastating. Among the damage that is caused, the disruption of the communication

P. Maya Duque (✉) · K. Sörensen
Faculty of Applied Economics, Operations Research Group ANT/OR, University of Antwerp, Stadscampus S.B. 513, Prinsstraat 13, 2000 Antwerp, Belgium
e-mail: pablo.mayaduque@ua.ac.be

P. Maya Duque
Faculty of Engineering, Universidad de Antioquía, Medellín, Colombia

and transport infrastructure deserves particular attention because it can obstruct the different phases of the disaster management and increase the loss of lives. This is particularly the case in developing countries where the infrastructure and communities are more vulnerable and sensitive to the impact of natural disasters (Twigg 2004). In this paper, we concentrate on the transport infrastructure, particularly on the repair plan for the rural road network in developing countries after the occurrence of a natural disaster (or a man-made disaster with similar consequences).

According to the UN International Strategy for Disaster Reduction, UN/ISDR (2002), disaster management activities can be categorized into five generic phases: prediction, warning, emergency relief, rehabilitation, and reconstruction. The last three phases are generally associated with the post-disaster effort and involve both response and recovery activities (Moe and Pathranarakul 2006). The response activities during the emergency relief phase aim for the provision of assistance during or immediately after a disaster to ensure the preservation of life and of basic subsistence needs of the affected people. Activities during the rehabilitation and reconstruction phases include decisions and actions taken after a disaster in order to restore or improve the living conditions of the affected community, but also activities related to mitigation and preparedness.

In this paper, we focus on the repair plan for the road network during the rehabilitation and reconstruction phases. Given a road network where some links have been damaged by a natural disaster, our aim is to determine how the scarce resources available for repair must be assigned in order to optimize the network's performance with respect to a selected objective, respecting budget and time limitations.

The objective we consider is *accessibility*, which is defined by Donnges (2003) as *the degree of difficulty people or communities have in accessing locations for satisfying their basic social and economic needs*. Lebo and Schelling (2001) define basic access as *the minimum level of rural transport infrastructure* (*RTI*) *network service required to sustain socioeconomic activity*. In this paper, we model the accessibility of a town as the *travel time required to reach the nearest* regional center. For any specific town, this travel time is found by calculating the lengths (measured in units of time) of the shortest paths to all regional centers and taking the minimum. Of course, this requires a substantial computational effort. In our algorithm, we therefore take great care to perform these calculations in an efficient way. The aim of our model is to support the design of a network repair plan that maximizes the accessibility considering the financial budget and number of hours of manpower available. In spite of operating on an abstract version of reality and a simplified version of the road network this kind of model can provide useful information and help to understand the problem better.

The problem we solve in this paper is defined on a graph that consist of a set of nodes, representing population centers and road cross points, and an underlying road network that connects those nodes. The set of nodes can be partitioned into three subsets: (i) a set of regional centers, usually composed of a small group of larger or more developed cities, (ii) a set of secondary cities, towns or villages, and (iii) a set of road cross points. The edges in the road network can be classified into two categories: (i) operative roads and (ii) damaged roads. Figure 1 shows an example of a typical network. The shadow region around each node represents the importance or weight of the associated city or town (e.g., the number of inhabitants).

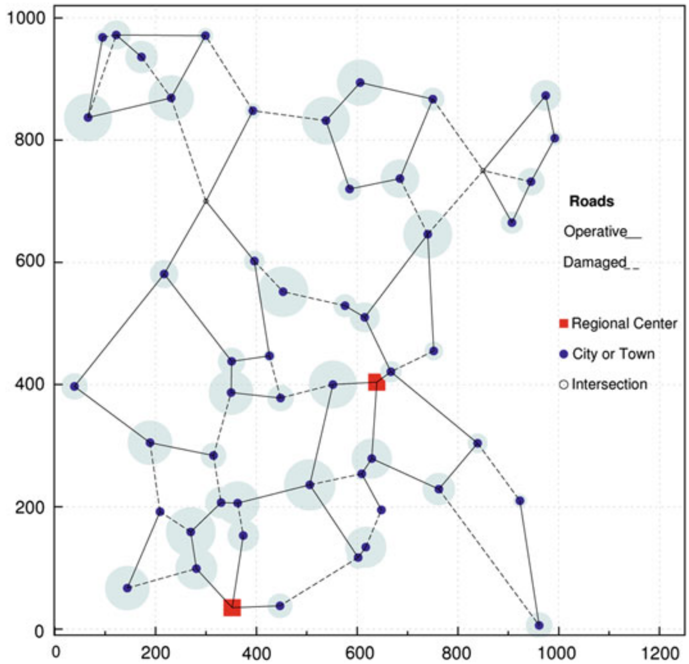

Fig. 1 An example of a road network. The size of a *circle* around a population center represents its relative importance

In an ideal scenario, all damaged roads would be repaired. However, due to constraints on the availability of economic resources and manpower, this configuration is generally not achievable within a short time period. Therefore, the objective of our problem is to look for a repair plan that maximizes the accessibility of the rural villages to the regional centers by repairing roads that are not operative after the natural disaster such that budget and manpower limits are kept. We also consider the fact that not all rural villages are of equal size, and that there is a preference to connect as many people as possible to the regional centers. We therefore attach an *importance factor* to each population center, e.g., its population size.

The problem of deciding which roads to upgrade gives rise to a complex optimization problem that features a highly nonlinear objective function and that cannot be readily solved by commercial solver software. In this section, we propose a mathematical programming formulation in which the non-linearity is explicitly stated. The notation used in that model is presented in Table 1. Let $\mathcal{G} = (\mathcal{N}, \mathcal{E})$ be an undirected graph where $\mathcal{N} = \{\mathcal{N}_1 \cup \mathcal{N}_2 \cup \mathcal{N}_3\}$ is a node set and $\mathcal{E}$ is an edge set. Edges in $\mathcal{E}$ represent roads and each of them has associated a level l_e that is 1 if the road is operative and 0 otherwise. The subset of edges $\mathcal{E}_r$ is composed of all roads that are not operative and can be repaired, for these roads the initial level is 0. A financial budget B and a manpower-time budget H are available in order to repair some roads. A financial cost c_e and a manpower requirement m_e are associated with each road $e \in \mathcal{E}_r$.

For each node i in $\mathcal{N}_2$ a measure of the accessibility is defined: the shortest travel time from i to the closest regional center in $\mathcal{N}_1$. Of course, this depends on the roads

Table 1 Symbols used in the mathematical programming formulation for the accessibility problem

Notation	
$\mathcal{N}$	Set of nodes of the graph on which the accessibility problem is defined Can be partitioned in three subsets: $\mathcal{N}_1$ regional centers, $\mathcal{N}_2$ rural towns and $\mathcal{N}_3$ road cross points
$\mathcal{E}$	Set of edges or roads Subset $\mathcal{E}_r \subset \mathcal{E}$ contains the roads that can be repaired
w_i	Weight (e.g., number of inhabitants) of node i
l_e	Status of the edge or road e (0: unrepaired, 1: repaired)
t_e	Time required to traverse road e
M_e	Extra time required to traverse road e when it is damaged
c_e	Cost to repair road e
m_e	Manpower hours required to repair road e
B	Financial budget
H	Manpower budget (in hours)

chosen to be repaired. The time to traverse an edge is t_e when the road is operative and $t_e + M_e$ when it is not. M_e represents a penalty factor for using another means to traverse e (e.g., by using animal-powered transport). A weight w_i is defined for each node i in $\mathcal{N}_2$ to represent the "importance" of the node. The value of w_i is therefore usually a function of the number of inhabitants of the rural town associated with node i. The objective is to minimize the weighted sum of the time to travel from each node i in $\mathcal{N}_2$ to its closest regional center in $\mathcal{N}_1$.

In our model, we do not explicitly take the capacity of the roads into account. If necessary, possible congestion on the roads can be taken into account by incorporating it in the travel time. In the case of rural or tertiary roads, which in developing countries are usually characterized by a low traffic level but a high connectivity importance, congestion (and therefore road capacity) is not an issue.

The binary decision variables x_e indicate whether road $e \in \mathcal{E}_r$ is repaired ($x_e = 1$) or not ($x_e = 0$). Additionally, two more decision variables are defined. Variable y_e^{ij} is assigned value 1 when the road e is used on the path from i to j and 0 otherwise. Similarly, variable b_k^{ij} is given value 1 when node k is visited on the path from i to j. The set of edges adjacent to node k is denoted by $\mathcal{E}(k)$. A mathematical integer program for the problem considered, based on the work done by Campbell et al. (2006) is the following:

$$\min \sum_{i \in \mathcal{N}_2} \left(w_i \min_{j \in \mathcal{N}_1} \left\{ \sum_{e \in \mathcal{E}} d_e y_e^{ij} \right\} \right) \tag{1}$$

$$d_e = \begin{cases} t_e + (1 - x_e) M_e, & \forall e \in \mathcal{E}_r \\ t_e, & \forall e \in \mathcal{E} \backslash \mathcal{E}_r \end{cases} \tag{2}$$

$$\sum_{e \in \mathcal{E}(i)} y_e^{ij} = 1 \quad \forall i \in \mathcal{N}_2, \forall j \in \mathcal{N}_1 \tag{3}$$

$$\sum_{e \in \mathcal{E}(j)} y_e^{ij} = 1 \quad \forall i \in \mathcal{N}_2, \forall j \in \mathcal{N}_1 \tag{4}$$

$$\sum_{e \in \mathcal{E}(k)} y_e^{ij} = 2b_k^{ij} \quad \forall k \in \mathcal{N} \backslash \{i, j\}, \forall i \in \mathcal{N}_2, \forall j \in \mathcal{N}_1 \tag{5}$$

$$\sum_{e \in \mathcal{E}_r} c_e x_e \leq B \tag{6}$$

$$\sum_{e \in \mathcal{E}_r} m_e x_e \leq H \tag{7}$$

$$x_e \in \{0, 1\} \quad \forall e \in \mathcal{E}_r \tag{8}$$

$$y_e^{ij} \in \{0, 1\} \quad \forall e \in \mathcal{E}_r, \quad \forall i \in \mathcal{N}_2, \quad \forall j \in \mathcal{N}_1 \tag{9}$$

$$b_k^{ij} \in \{0, 1\} \quad \forall k \in \mathcal{N} \backslash \{i, j\}, \quad \forall i \in \mathcal{N}_2, \quad \forall j \in \mathcal{N}_1 \tag{10}$$

The objective function (1) minimizes the weighted sum of the shortest paths for all $i \in \mathcal{N}_2$ to the nearest regional center $j \in \mathcal{N}_1$. Constraints (2) determine the travel time for road e given the upgrade decision for the roads. Constraints (3) and (4) enforce, respectively, that there is exactly one edge leaving i on the path from i to j, and that there is exactly one edge entering j on the path from i to j. Constraints (5) ensure that the path from i to j is connected. Constraints (6) and (7) define the budget limitations. The decision variables are defined to be binary in constraints (8), (9) and (10).

By defining $\mathrm{SP}_{ij}(\mathbf{x})$ as the travel time on the shortest path from i to j given a solution vector $\mathbf{x}$ for the roads to be repaired, the problem can be reformulated according to a more intuitive model, as follows:

$$\min \sum_{i \in \mathcal{N}_2} \left(w_i \min_{j \in \mathcal{N}_1} \{\mathrm{SP}_{ij}(\mathbf{x})\} \right) \tag{11}$$

$$\sum_{e \in \mathcal{E}_r} c_e x_e \leq B \tag{12}$$

$$\sum_{e \in \mathcal{E}_r} m_e x_e \leq H \tag{13}$$

$$x_e \in \{0, 1\} \quad \forall e \in \mathcal{E}_r \tag{14}$$

This paper is organized as follows. Section 2 surveys the relevant literature on related problems. Our solution strategy is proposed in Sect. 3. Tests to validate the algorithm performance are carried out in Sect. 4. Finally, Sect. 5 contains some conclusions and pointers for future research.

2 Literature review

Disaster management is an upcoming area in operations research and it has the potential to make a significant impact on society as is pointed out by Ergun et al. (2010). Van Wassenhove (2003) affirms that “the subject of disaster management is an absolutely fascinating one that is growing in importance”. However, various authors agree that disaster management is a topic in which the OR/MS community still has to strengthen its critical mass.

Altay and Green (2006) and Ergun et al. (2010) highlight that disaster recovery is one of the areas that need more research, and state that research on this topic is abundant in other fields but not in OR/MS. They point out research directions such as: damage assessment and cleanup, monetary aid collection and allocation, and distribution problems. The problem we address in this paper falls within those research directions.

The problem of allocating resources in a repair plan for the road network after the occurrence of a natural disaster can be modeled as a discrete network design problem (DNDP). However, the accessibility objective must be considered instead of the total travel cost in the network. The accessibility-maximization approach has been considered in the road network planning model by Antunes et al. (2003). Their approach is based on a non-linear combinatorial optimization model, and two heuristics are used to solve the model. That model does not involve an evolution of the network over time but defines the final status of the network at the end of the planning horizon. Scaparra and Church (2005) also tackle the road network planning problem, focusing on the rural case for developing countries. The authors propose a GRASP (Greedy Randomized Adaptive Search Procedure) and path relinking heuristic, and consider a bi-objective model, which minimizes the sum of the weighted shortest paths between all pairs of nodes and maximizes the traffic flow. The maximal covering network improvement problem is studied by Murawski and Church (2009) with the objective of improving accessibility to rural health service. The problem is formulated as an integer linear programming problem, and is applied to a real case in the Suhum District of Ghana. The convenience of using a metaheuristic approach for larger instances is pointed out by the authors.

The reconstruction or repair of the road network after a natural disaster has been studied by several authors considering different objectives and using diverse approaches. Chen and Tzeng (1999) propose a genetic algorithm to solve a fuzzy multi-objective model of a road network reconstruction effort. Their model consists of two levels: an upper level that schedules the work teams and a lower level that is an asymmetric traffic assignment model. Three objectives are considered: minimizing the total travel time over the road network during reconstruction, minimizing the individual reconstruction time of any work team, and minimizing the idle time between work teams. A multi-criteria model for planning post-earthquake sustainable reconstruction is presented by Opricovic and Tzeng (2002) and its applicability is illustrated in a real case (Chi Chi-Taiwan earthquake) for the restoration of lifeline systems such as electricity, water, and transportation networks. Feng and Wang (2003) focus on the network repair over the first 72 h after the disaster, i.e., as part of the response activities. The authors propose a multi-objective programming model to maximize the performance of emergency road rehabilitation, maximize the number of people benefited, and minimize the risk for rescuers. A case study based in the disaster occasioned by an earthquake in Taiwan is presented. Karlaftis et al. (2007) develop a methodology to optimally prioritize bridge repair in an urban transport network following a natural disaster. The resource allocation problem is formulated as a three-stage model and a genetic algorithm is implemented to solve it. Recently, Yan and Shih (2009) study emergency repair and relief distribution planning from an integrated point of view.

They employ network flow techniques to develop a multi-objective integer programming model and propose a heuristic to solve it.

The study of upgrading a network has been addressed before by several authors in the field of network design problems. *Budget constrained network upgrading problems* are studied by Krumke et al. (1998b). The authors define two different variants of the problem: the edge-based upgrading model and the node-based upgrading model. In both cases, they consider two objectives f_1 (the cost of the network) and f_2 (an objective related to the quality of the network), and a membership requirement in a class of subgraphs $\mathcal{S}$ (e.g., the set of spanning trees). The problem defines a budget value on the first objective f_1, and the goal is to find a network that has minimum possible value for the second objective f_2, such that this network is within the budget on f_1 and belongs to the subgraph-class $\mathcal{S}$. As a particular case the functions are defined in such a way that the cost of improving the network is limited to a budget and the total length of a minimum spanning tree has to be minimized. For the edge-based upgrading model, three cases are considered depending on the values to which each edge can be upgraded: binary (upgraded, not upgraded), integer (several upgrading levels), rational (rational values in an interval). For binary upgrades, minimization of the total weight of the graph under the budget constraint is *NP-hard* even for trees. Some results on the complexity and approachability and some algorithms for special cases are presented in Krumke et al. (1998a).

Campbell et al. (2006) define the *q-arc upgrading problem* that involves finding the best q arcs in a network to upgrade. Two cases of this problem are studied. The *q-upgrading arc diameter problem* selects q arcs to upgrade such that the travel time on the maximum shortest path between any origin–destination pair (i.e., the diameter of the network) is minimized. The *q-upgrading arc radius problem* selects q arcs to upgrade and locates the vertex center, i.e., the node for which the maximum shortest path to the other nodes in the network (i.e., the radius of the network) is minimized. The authors show that those problems are *NP-hard* on general graphs, but polynomially solvable on trees. A variant of the problem which considers a budget constraint is also studied, it is shown that this problem is *NP-hard* for general graphs and even for a path graph. Three heuristic algorithms are proposed to deal with this kind of problems.

GRASP is defined by Resende and Ribeiro (2010) as a multi-start or iterative metaheuristic, in which each iteration consists of two phases: construction and local search. The construction phase builds a solution. Once a feasible solution is obtained, its neighborhood is investigated until a local minimum is found during the local search phase. The best overall solution is kept as the result. As is pointed out by Festa and Resende (2009), GRASP has been applied successfully to many problems in different areas, such as scheduling, routing, logic, partitioning, location, graph theory, assignment, manufacturing, transportation, telecommunications, biology and related fields, automatic drawing, power systems, among others. One of the main characteristics of GRASP is its simplicity of both implementation and comprehension that makes it easy to communicate it to the users. GRASP makes use of simple building blocks (solution construction procedures and local search methods) and its basic version requires the adjustment of only a single parameter (Resende and Ribeiro 2010).

In the basic GRASP heuristic almost all the randomization effort involves the construction phase. Strategies such as variable neighborhood search (VNS), rely almost entirely on the randomization of the local search to escape from local optima. In this context, Resende and Ribeiro (2010) have pointed out that GRASP and variable neighborhood strategies may be considered as complementary and potentially capable of leading to effective hybrid methods.

3 A metaheuristic for the accessibility problem

Immediately after a natural or man-made disaster has destroyed parts of the road infrastructure, a decision must be taken which roads to repair, so as to maximize the accessibility. In this section we present a metaheuristic to solve this problem. Our approach is essentially a GRASP and consists of an iteration of two phases: a greedy randomized construction phase, followed by a neighborhood-search improvement phase.

To define our metaheuristic we use the same notation we introduced when presenting the mathematical formulation. Additionally, we define e^- and e^+ to be the vertices connected by edge e, and the function $f_t(e, l)$ that gives the time to traverse edge e when it is at level l.

The objective function of our metaheuristic requires the calculation of the shortest path between each rural village (nodes in $\mathcal{N}_2$) and its nearest regional center (nodes in $\mathcal{N}_1$). To keep the computational cost of calculating the objective function to a minimum, we take particular care in efficiently storing the solution and efficiently updating the objective function after each insertion of the construction phase or after each move of the improvement heuristic. In the proposed approach the construction phase uses an *insertion algorithm* (IA) while the improvement or search phase corresponds to a variant of the *variable neighborhood search* (VNS). These two phases, as well as the general structure of the metaheuristic are presented in the following subsections.

3.1 Solution representation

A full description of a feasible solution for this problem, that allows us to calculate the objective function, does not only store the set of roads that should be repaired but also the length of the shortest path from each node i in $\mathcal{N}_2$ to its nearest regional center j in $\mathcal{N}_1$. To efficiently update the solution after a change requires us to store also the sequence of each of these shortest paths. To represent a feasible solution we therefore use two square matrices: the shortest path length ($\mathbf{T}$) and the shortest path sequence ($\mathbf{P}$). Element $T[i, j]$ represents the shortest travel time from i to j. Element $P[i, j]$ keeps the first node of the shortest path from i to j, allowing to trace the complete path recursively.

3.2 Construction phase: initial solution heuristic

As pointed out before, the heuristic to build an initial feasible solution is based on an insertion algorithm. This algorithm starts with an empty solution $\mathbf{x} = \mathbf{0}$ where each road in $\mathcal{E}_r$ is preserved in its current unrepaired status and the remaining budgets $\widehat{B}$ and $\widehat{H}$ are fixed to their initial values B and H. At the start of the insertion algorithm, matrices $\mathbf{T}$ and $\mathbf{P}$ are computed using the Floyd–Warshall algorithm described in Hu and Shing (2002). The computational complexity of the Floyd–Warshall algorithm to compute the shortest path between all pairs of nodes in the initial road network is $\mathcal{O}(n^3)$.

The insertion algorithm then iteratively repairs some roads in order to improve the total accessibility and continues until no more improvements can be made within the remaining budgets. A schematic overview of the insertion algorithm is given in Algorithm 1.

Algorithm 1 Insertion algorithm

Initialization: Consider initial $\mathbf{x}$, $\widehat{B}$, $\widehat{H}$, $\mathbf{T}$, and $\mathbf{P}$
repeat
  $saving = 0$
  for $\left(e \in \mathcal{E}_r \;\mid\; l_e \neq 1 \wedge c_e \leq \widehat{B} \wedge m_e \leq \widehat{H}\right)$ **do**
    Estimate insertion saving for e, $saving(e)$
    if $saving(e) > saving$ **then**
      $saving = saving(e)$
      $candidate = e$
    end if
  end for
  Improve $candidate$: Update $\mathbf{x}$, $\widehat{B}$, $\widehat{H}$, $\mathbf{T}$, and $\mathbf{P}$; $saving = 0$
until no insertion candidate is found

The shortest path recalculation can be required for estimating the insertion saving and updating $\mathbf{T}$ and $\mathbf{P}$. However, in order to decrease the computational effort, these tasks are performed in an efficient way. The saving achieved by improving the road e, that connects vertices i and j, can be computed exactly without new shortest path calculations as follows:

$$\text{saving}(e) = \sum_{l \in \mathcal{N}_2} w_l \left(\min_{k \in \mathcal{N}_1} \{T[k,l]\} - \min_{k \in \mathcal{N}_1} \{T[k,i] + f_t(e,1) + T[j,l]\} \right) \tag{15}$$

As the roads are considered undirected, the saving should also consider to traverse e in direction $j \longrightarrow i$. However, if one of the two directions in which the road can be traversed provides a path shorter than the other one, the saving is computed with respect to that path.

After upgrading a road it is necessary to update $\mathbf{T}$ and $\mathbf{P}$. A procedure to recalculate the shortest paths has been implemented that avoids us having to compute them from scratch. That procedure evaluates for every pair of nodes if the path connecting them and traversing e, at its new status, is shorter than the current shortest path. The computational complexity of this procedure is $\mathcal{O}(n^2)$.

The insertion algorithm described in this section is completely deterministic. It therefore generates the same initial solution each time it is restarted. To be usable in our GRASP metaheuristic however, it must be able to provide various initial solutions. We therefore modify Algorithm 1 to select the road to repair randomly from a restricted candidate list (RCL). The RCL contains the α best possible insertions one of which is selected randomly. The length of that list α is a parameter of the algorithm.

3.3 Improving phase: variable neighborhood descent

The improving phase is based on the *variable neighborhood search* metaheuristic and corresponds to the variable neighborhood descent (VND) method described in Hansen and Mladenovic (2005). Algorithm 2 presents an overview of the VND implemented for the improving stage.

Algorithm 2 Improving phase

Initialization: Consider an initial solution $\mathbf{x}$, Select the set of neighborhood structures $N_k(\mathbf{x})$ $k = 1, 2, \ldots, k_{\max}$.
Set $k \longleftarrow 1$
repeat
Explore Neighborhood k. Find the best neighbor $\mathbf{x}'$ of $\mathbf{x}$ ($\mathbf{x}' \in N_k(\mathbf{x})$)
Move or not. If the solution $\mathbf{x}'$ thus obtained is better than $\mathbf{x}$, set $\mathbf{x} \longleftarrow \mathbf{x}'$ and $k \longleftarrow 1$; otherwise, set $k \longleftarrow k + 1$;
until $k > k_{\max}$

Our VND algorithm uses three move types, defining three different neighborhood structures ($k_{\max} = 3$). First, downgrade changes the status of a road from repaired (1) to damaged (0). Second, the upgrade move improves the status of a road from damaged to repaired. Incidentally, this is the move that is used by the insertion algorithm to build the initial solution. The third move, swap, simultaneously applies the two previous moves, i.e., it changes a road from damaged to repaired and another one from repaired to damaged. The neighborhood structures are used in the order suggested by Scaparra and Church (2005), i.e., (1) downgrade, (2) swap, and (3) upgrade.

The neighborhood defined by the downgrade move never contains a solution with a better objective function value than the current solution. In the best case, downgrading a road does not cause an increase in any of the shortest paths in the objective function. However, it can be possible to find neighbors with the same objective function value but that use less resources (i.e., less of the available budgets). By applying the search in this neighborhood repetitively, a solution with less useless road improvement can potentially be found. The downgrade move is used in the following way. For each road $e \in \mathcal{E}_r$ we determine whether or not it is part of any of the shortest paths $\mathrm{SP}_k(O_k)$ that connect each node $k \in \mathcal{N}_2$ to its nearest regional center $O_k \in \mathcal{N}_1$. If this is not the case, road e is downgraded to its original status without affecting the current value of the objective function. In order to check this condition matrix $\mathbf{P}$ is used. It is possible to show that a road e that connects vertices i and j is part of at least one shortest path $\mathrm{SP}_k(O_k)$, $k \in \mathcal{N}_2$, if:

$$P[i, O_i] = j \quad \text{or} \quad P[j, O_j] = i \tag{16}$$

Each road e that can be downgraded is set to its initial status. In this way, a solution with the same objective value but with less resources usage is obtained. The procedure has complexity $\mathcal{O}(|\mathcal{E}_r|)$.

To update matrices **T** and **P**, after downgrading edge e, the *small label first* (SLF) algorithm for the shortest path problem is used. This algorithm belongs to the class of label-correcting methods which are more suitable for re-optimizing than the label-setting methods (e.g., Dijkstra's algorithm), as they allow to use advanced initializations and exploit the particularities of sparse graphs. Bertsekas (1998) presents an extensive discussion on this topic. The re-optimization is performed for all nodes that use edge e in their shortest path tree.

The second neighborhood is defined by the upgrade move and the search on it is performed as was described in Sect. 3.3 when an initial solution was constructed.

The third neighborhood (swap) considers all neighbor solutions that can be reached from the current solution by simultaneously upgrading one road and downgrading another, while not violating any constraints. To compute the saving generated with this move we must sum up the effect of the move over the shortest path from each $i \in \mathcal{N}_2$ to its nearest regional center. This can require recalculating all shortest paths. However, in order to reduce the computational time we opt to estimate the swap saving without new shortest path calculations as follows:

$$\text{saving}(e) = \sum_{l \in \mathcal{N}_2} w_l \times \left(T\,[l,\, O_l] - \min_{k \in \mathcal{N}_1} \left\{ \widehat{T[l,k]} \right\} \right) \tag{17}$$

Where $T[l, O_l]$ is the current shortest path from l to its closest regional center O_l, while $\widehat{T[l,k]}$ estimates the travel time from l to regional center k after the swap move. Let Δe be the absolute value of the change in travel time over the road e, with respect to its current status. Let e^{u} be the road that is upgraded and e^{d} be the road that is downgraded. When a swap move is performed, three cases are possible:

1. The path from l to k traverses the upgraded edge e^{u} but not the downgraded edge e^{d} In that case $\widehat{T[l,k]} = T[l,k] - \Delta e^{\mathrm{u}}$ represents the improvement in travel time from l to k.
2. The path from l to k traverses the downgraded edge e^{d} but not the upgraded edge e^{u}. In that case $\widehat{T[l,k]} = T[l,k] + \Delta e^{\mathrm{d}}$ can overestimate the effect of the swap move, because the new shortest path from l to k could avoid traversing e^{d}.
3. The path from l to k traverses both the downgraded edge e^{d} and the upgraded edge e^{u}. In that case $\widehat{T[l,k]} = T[l,k] - \Delta e^{\mathrm{u}} + \Delta e^{\mathrm{d}}$ also can overestimate the effect of the swap move.

When a large constant M is used to define M_e for all roads, a swap candidate generates a positive saving estimation only when it has a large effect in improving the accessibility of nodes with large importance w_l. As a consequence in many cases no profitable candidates to swap are found. For this reason, we consider a second criterion to select swap candidates that states that the status of two roads can be swapped when the sum of the populations of the towns for which their shortest path to the nearest regional center traverses e^{u} (case 1) is greater than the sum of the populations of the towns for which their shortest path to the nearest regional center traverses e^{d} (case 2). However, this criterion allows moves that deteriorate the value of the objective function and can generate cycles. In order to avoid such possible cycling we introduce a parameter to control the number of times that the criterion can be used.

Matrices **T** and **P** are updated in two steps. First, the updating procedure is applied considering only the upgrading of the edge e^{u}. Then, the SLF algorithm described for the *downgrade* move is applied considering only the downgrading of the edge e^{d}.

3.4 Extensions

The algorithm described in this section is designed to deal with a simplified model of the real-life problem. This is largely due to the complexity of the problem and the kind of decision process to which it is oriented. Moreover, data availability (e.g., on the cost of repairing certain roads) is usually extremely limited in rural areas of third-world countries and uncertain estimates with large margins of error need to be made. For these reasons, a simple and robust model that generates information to support the decision process is more useful than a complex and untractable model that tries to incorporate every real-life aspect. However, it is important to point out how this model can be extended to deal with some other characteristics of the real problem.

In this paper we assume that after the disaster the roads are restored to the condition they were in before the disaster. However, the roads may also be partially repaired (e.g., to save resources) which may lead to a longer traveling time than in the original state. In addition, roads that were already in a bad condition may be upgraded to a better level than before the disaster. This would require modifications to both the model and the solution algorithm. The model in this case also demands more information, because an estimation of the cost and travel time for each road in each possible status is required. Such information is generally not available in developing countries and extremely difficult to gather. We have therefore opted to consider only the binary case (in which a road is either repaired or not) in this paper.

Additionally, it is reasonable to consider alternative locations to reconstruct a road that has been damaged by a disaster. In the case of a road struck by a flooded river, for example, it may be a good idea to build another road further away from the river to avoid future flooding. This kind of decision can be readily handled by the model and algorithm in this paper by defining "dummy roads" that do not exist yet (and therefore have infinite travel time). "Repairing" such a road is in this case equivalent to building it, after which the road will have a finite travel time.

4 Computational results

In this section, we report the results from a computational study to evaluate the performance of our metaheuristic approach. In a first phase we solve several small instances and compare the results to the optimal solutions to demonstrate the excellent performance of our approach. Although most solvers can, in principle, solve non-linear problems, they require (at least for the ones we are aware of) that the constraint matrix is positive semi-definite. This is not guaranteed for our problem. We have therefore purpose-built an exact solution method for this specific application, essentially an intelligent complete enumeration approach. Our procedure recursively builds a tree of solutions in which the nodes at level n represent trees that have exactly n repaired roads. The root of the tree is a solution with all roads at their current level. The procedure uses a depth-first search, and repairs one road in the current solution at each iteration, going one level down in the tree. When the budget or manpower constraint is violated by adding a road, the procedure backtracks and continues with another branch of the tree one level up. Developing a more elaborate exact solution method is beyond the scope of this paper, and will probably be of limited use in a practical disaster response situation for the reasons outlined in Sect. 3.4. Unfortunately, our exact approach is only able to solve small instances as the computing time it requires rises exponentially with the size of the problem. In a second phase, we test our algorithm on a large instance partially based on the (limited) information available for a natural disaster that occurred in Haiti in 2008. Our enumerative procedure and GRASP/VND metaheuristic have been implemented in Java and all experiments were performed on a computer with a 2.50 GHz dual core Intel processor and 3.5 GB RAM. All data sets are available from the http://antor.ua.ac.be/downloads.

4.1 Randomly generated instances

In a first set of experiments, we concentrate on determining the effectiveness and efficiency of the metaheuristic proposed in this paper. In order to do so, we have generated several small random instances with two regional centers, 40 or 50 towns, 3 road cross points, and between 55 and 75 roads of which at most 33% are damaged.

Instances have been generated according to two distinct network topologies, that we call the *branching* pattern and the *general* pattern respectively. Instances generated following the branching pattern, are characterized by the fact that most of the nodes corresponding to towns (nodes in $\mathcal{N}_2$) have degree one or two, a pattern that closely resembles the real-life situation of most rural road networks. Instances that follow the general pattern do not have this property. For each instance, the coordinates of the different nodes are generated using a $U[0, 1000]$ distribution and rounded to the nearest integer. The weight associated with each town follows a $U[20, 70]$ distribution. The roads are created by hand in order to obtain the desired pattern. The cost and repair time for each damaged road are generated as a linear function of its length. The budgets are set as a percentage of the total budget required to generate all roads. Three levels are considered: 15, 30, and 60%.

In total 90 experiments were defined, as a result of considering 10 instances and 9 different combinations of budget levels for each instance. The computational time for the enumerative algorithm ranges between a few seconds and 27 h depending on the instance and the combination of the budget levels. For each experiment, 1000 iterations of our metaheuristic were executed and the best solution was kept. Table 2 presents a summary of the results for those experiments.

The metaheuristic provides the optimal solution for 85 out of 90 experiments. For all the instances following the general pattern, the optimal solution was found. For only five experiments with the branching pattern, the metaheuristic could not find the

Table 2 Computational results for small simulated instances

Pattern	Branching	General	Total
Number of experiments	45	45	90
Optimal solutions	40	45	85
Gap to optimality (%)			
Maximum	2.7	0.0	2.7
Average	0.2	0.0	0.1
Computational effort (s)			
Maximum	7.6	5.9	7.6
Average	4.6	4.1	4.3
Iterations to best solution			
Maximum	422	552	552
Average	90.6	114.0	102.3

optimal solution, with a maximum optimality gap of 2.7%. Computing time was <8 s for all the experiments and, on average, not more than 103 iterations were required to obtain the best solution.

4.2 Real-life-based instance

To test our metaheuristic under more realistic conditions, we have built an instance based on the information gathered from diverse sources for a real natural disaster that occurred in Haiti.

Within weeks at the end of August and the beginning of September 2008, four hurricanes and tropical storms hit Haiti. According to the official reports OCHA-UN (2008); OMP (2009) up to 800,000 people were directly affected and many main roads and bridges across the country were destroyed or blocked, compounding logistics operations. We have generated an instance based on this case with the information we obtained from diverse sources. Data from *GISDataDepot* (http://data.geocomm.com) was used to define the network in a Geographical Information System (GIS). The status of the network after the natural disaster was defined based on the information published by *Mapaction* (www.mapaction.org) and *Reliefweb* (www.reliefweb.com). Demographic information was obtained from *Falling Rain Genomics Inc.* (www.fallingrain.com). The cost and time required to repair each road were estimated as a linear function of its length because it was not possible to access these parameters directly.

The instance generated in this way has 216 nodes (103 of them representing cities or towns) and 281 roads (30 of them damaged after the disaster). Figure 2 depicts the network, labels for some of the most important cities have been added.

We selected three cities as the regional centers (Port Au-Prince, Les Cayes, and Cap Haitien) and used the network described before to define the conditions after the occurrence of the natural disaster. Based on these conditions, we define three different scenarios in which 25, 50, and 75% of the total monetary and manpower budgets required to repair all roads is available respectively. The penalization M_e for using a different transport mean to traverse a road when it is not operative is set to the same value for all $e \in \mathcal{E}_r$. The value we use is the sum of the length of all the roads in $\mathcal{E}_r$.

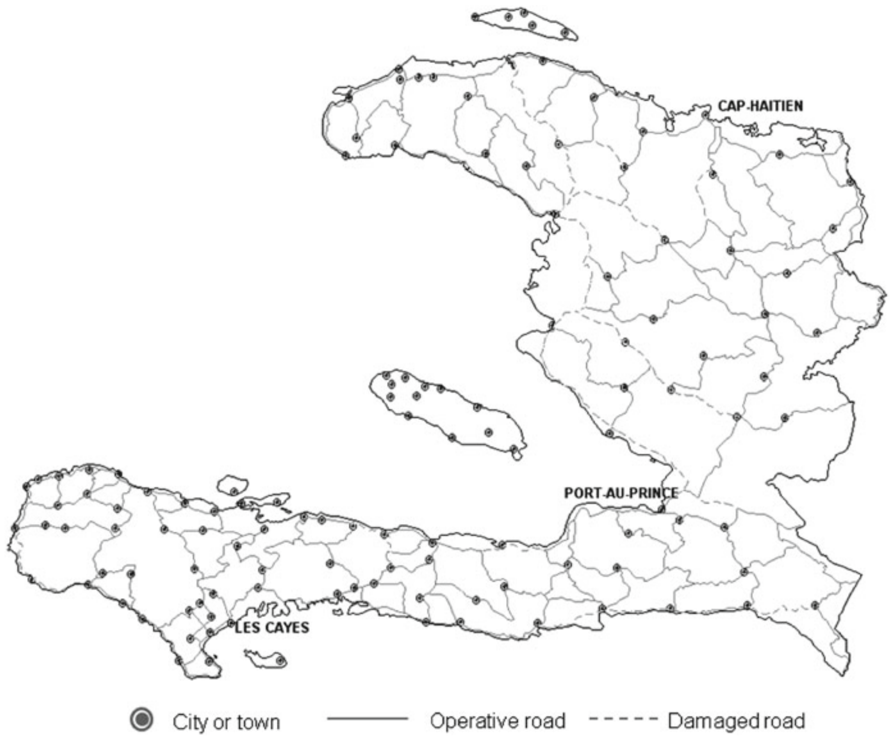

Fig. 2 Road network after the natural disaster

Table 3 summarizes the conditions just after the occurrence of the disaster (column Initial) and for the three budget scenarios defined. The affected towns and people (for which the shortest path to their nearest regional center is either longer than before, or non-existent) are classified as *accessible* when there exists a path to connect them (town or people) to a regional center but that path is longer than it was before the disaster. They are classified as *not accessible* when there is no path connecting them to a regional center. For each town the difference in travel time between the shortest path to the nearest regional center before and after the disaster is computed. After repairing the network, we determine how much this difference has been reduced and the average of that measurement for all the towns affected is presented as a percentage in the row denoted by *Avg. recovery on SP*.

The final solution produced by our metaheuristic shows that when a high value of M_e is used for all $e \in \mathcal{E}_r$, it gives priority to the inaccessible towns. For all scenarios the resources are allocated such that after the network has been repaired there are no disconnected towns. When the budgets are set to half of the total requirements, Fig. 3, our metaheuristic provides a solution which has recovered almost 97% of the accessibility with respect to the conditions before the disaster. This result clearly shows that a good planning of the roads is important. In that solution only one town (Anse d'Hainault) remains affected in terms of its accessibility as shown in Table 3. With three quarters of the total requirements in terms of budget, the network can be totally

Table 3 Computational results for the Haiti case

	Initial		25–25		50–50		75–75	
Towns affected	31		14		1		0	
Accessible	13		14		1		0	
Not accessible	18		0		0		0	
Avg. recovery on SP (%)	–		58.1		96.8		100.0	
		%		%		%		%
People affected								
Total	774,432	14.8	398,631	7.6	8,163	0.2	0	0.0
Accessible	364,259	7.0	398,631	7.6	8,163	0.2	0	0.0
Not accessible	410,173	7.8	0	0.0	0	0.0	0	0.0

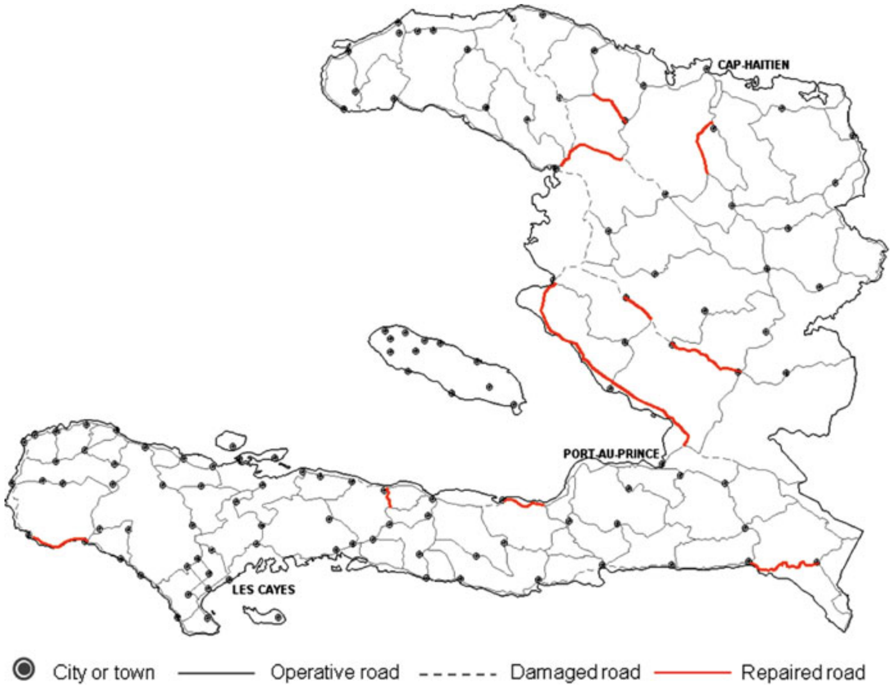

Fig. 3 Repaired road network when budgets are set to 50% of total requirement

recovered with respect to the accessibility. This does not mean that all roads have been repaired but, with the roads that have been repaired, the distance of each town to its nearest regional center is the same as it was before the disaster (Fig. 3).

As mentioned, M_e represents a penalization factor for using an alternative means of transport to traverse the road that connects i and j. In order to visualize the effect that this parameter has on the solutions obtained, experiments with different values for each damaged road were carried out. We have tested values of M_e that correspond to a situation in which travel times to traverse the damaged roads are double, triple, and ten times the original time. Results from this experiment show that this value has an important impact on the results obtained with our metaheuristic. When the value of the penalty is such that travel times are double or triple their original values, our metaheuristic does not prioritize accessibility to the disconnected towns but repairs the roads that have a large impact on the shortest paths. This results in final solutions in which some towns are not connected to a regional center, because the segments of the paths that are damaged do not impose a high penalty on the shortest path. When the penalty is set to a value such that travel times on the roads are ten times as high when the roads are damaged than when they are not, the metaheuristic connects all towns to a regional center. Therefore, these penalties can be used to include into the model the importance of each road, and they provide a way to deal with the objective of making all towns accessible, something which is not considered explicitly in the model.

5 Conclusions and future research

We have considered the problem of allocating resources for repairing a road network after a natural disaster in order to maximize the accessibility of the rural villages to the regional centers. A solution approach based on the GRASP and VND metaheuristics has been described and implemented. The efficiency of our approach was tested on a set of small and medium size instances and on a large instance constructed based on information gathered for a natural disaster that occurred in Haiti in 2008. The results demonstrate the excellent performance of our approach and point out the importance of using OR techniques to support the decision process in disaster management.

There are two potential directions for future research that arise from this work. First, the scope of the problem can be enlarged by considering multiple objectives and multiple periods. The multi-objective problem allows to consider, for example, the interaction between cost-efficiency and accessibility, while the multi-period version can spread the resource availability and the repair decisions over a time horizon. Secondly, we can consider the problem of repairing the road network within the emergency relief phase, i.e., the emergency repair of the network during the short period after the disaster. In this case, the scheduling and routing decisions of the work teams must be considered as part of the optimization problem.

References

Altay N, Green W (2006) OR/MS research in disaster operations management. Eur J Oper Res 175:475–493

Antunes A, Seco A, Pinto N (2003) An accessibility-maximization approach to road network planning. Comput-Aided Civil Infrastruct Eng 18:224–240

Bertsekas D (1998) Network optimization. Continuous and discrete models, chap 2. The shortest path problem. Athena Scientific, pp 51–114

Campbell A, Lowe T, Zhang L (2006) Upgrading arcs to minimize the maximum travel time in a network. Networks 47:72–80

Chen Y, Tzeng G (1999) A fuzzy multi-objective model for reconstructing post-earthquake road-network by genetic algorithm. Int J Fuzzy Syst 2:85–95

Donnges C (2003) Improving access in rural areas: guidelines for integratedrrural accessibility planning. Technical report, International Labour Organization

Ergun O, Karakus G, Keskinocak P, Swann J, Villarreal M (2010) Operations research to improve disaster supply chain management. Wiley Encyclopedia of Operations Research and Management Science, 8
Feng C, Wang T (2003) Highway emergency rehabilitation scheduling in post-earthquake 72 hours. J Eastern Asia Soc Transp Stud 5:3276–3285
Festa P, Resende M (2009) An annotated bibliography of GRASP. Part II: applications. Int Trans Oper Res 16:131–172
Hansen P, Mladenovic N (2005) Search methodologies. Introductory tutorials in optimization and decision support techniques. Variable neighborhood search. Springer, New York, pp 211–238
Hu TC, Shing MT (2002) Combinatorial algorithms, chap 1. Shortest paths. Dover, pp 10–16
Karlaftis M, Kepaptsoglou K, Lambropoulos S (2007) Fund allocation for transportation network recovery following natural disasters. J Urban Plan Dev 133:82–89
Krumke S, Marathe M, Noltemeier H, Ravi R, Ravi S (1998a) Approximation algorithms for certain network improvement problems. J Combin Optim 2:257–288
Krumke S, Marathe M, Noltemeier H, Ravi R, Ravi S (1998b) Network design: connectivity and facilities location. Network improvement problems. American Mathematical Society, pp 247–268
Lebo J, Schelling D (2001) Design and appraisal of rural transport infrastructure ensuring basic access for rural communities. Technical report, World Bank
Moe T, Pathranarakul P (2006) An integrated approach to natural disaster management: public project management and its critical success factors. Disaster Prev Manag 15:396–413
Murawski L, Church R (2009) Improving accessibility to rural health services: the maximal covering network improvement problem. Socio-Econ Plan Sci 43:102–110
OCHA-UN (2008) Haiti flash appeal 2008. Technical report. Office for the cordination of humanitarian affairs
OMP (2009) WFP-assisted development projects, protracted relief and emergency operations. Technical report. The Regional Bureau for Latin America and the Caribbean
Opricovic S, Tzeng G (2002) Multicriteria planning of post-earthquake sustainable reconstruction. Comput-Aided Civil Infrastruct Eng 17:211–220
Resende M, Ribeiro C (2010) Greedy randomized adaptive search procedures: advances and applications. In: Handbook of metaheuristics, 2nd edn. Springer, New York, pp 281–317
Scaparra M, Church R (2005) A GRASP and path relinking heuristic for rural road network development. J Heuristics 11:89–108
Twigg J (2004) Good practice review 9. Disaster risk reduction: mitigation and preparedness in aid programming. Overseas Development Institute
UN/ISDR (2002) Living with risk: a global review of disaster reduction initiatives. Preliminary version. Technical report, ISDR Secretariat
Van Wassenhove L (2003) New interesting POM cases from Europe. POMSChronicle 10:19
Yan S, Shih Y (2009) Optimal scheduling of emergency roadway repair and subsequent relief distribution. Comput Oper Res 36:2049–2065