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PROMETHEE and AHP: The design of operational synergies in multicriteria analysis. Strengthening PROMETHEE with ideas of AHP

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Abstract

This paper discusses the strengths and weaknesses of the Preference Ranking Organisation MeTHod for Enrichment Evaluations (PROMETHEE) and analytic hierarchy process (AHP) methods. Building upon this analysis, recommendations are formulated to integrate into PROMETHEE a number of useful AHP features, especially as regards the design of the decision-making hierarchy (ordering of goals, sub-goals, dimensions, criteria, projects, etc.) and the determination of weights. As a result of mixing basis features of both methods, operational synergies can be achieved in MCA.

Keywords: Multicriteria analysis; AHP; PROMETHEE

1. Introduction

This paper discusses two well known and widely used multicriteria analysis (MCA) methods, which were developed 20 years ago and have been used extensively since, for project and policy evaluation purposes. The first is Preference Ranking Organisation MeTHod for Enrichment Evaluations (PROMETHEE) developed by Brans (1982) and further extended by Brans and Vincke (1985); Brans and Mareschal (1994). The second is the analytic hierarchy process (AHP) method developed by Saaty (1982, 1988, 1995). The former is an outranking method, typical for the European (or French) MCA school. The latter builds upon complete aggregation of the additive type, characteristic of the American school.

Section 2 briefly describes the two methods. In Section 3, the strengths and weaknesses of both methods are analysed. Section 4 includes a number of recommendations to integrate into PROMETHEE a number of useful features, characteristic of the AHP method. These features are related to the design of a decision-making hierarchy and the determination of weights. In this context, a new method is proposed for determining the weights, building upon the mechanism of pairwise

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comparisons, which is characteristic of the AHP approach. This new method differs from the approach adopted by Hens et al. (1992), who merely applied the conventional AHP mechanism (without substantive modifications) to determine the weights in PROMETHEE-based MCAs. As a result of "mixing" the two MCA approaches, operational synergies can be achieved.

2. An overview of the PROMETHEE and AHP approaches

2.1. The PROMETHEE-approach

2.1.1. Principles

The evaluation table is the starting point of the PROMETHEE method. In this table, the alternatives are evaluated on the different criteria. These evaluations involve essentially numerical data.

The implementation of PROMETHEE requires two additional types of information, namely:

- information on the relative importance (i.e., the weights) of the criteria considered;
- information on the decision-maker's preference function, which he/she uses when comparing the contribution of the alternatives in terms of each separate criterion.

The above information determines the preference structure of the decision-maker.

2.1.2. The weights

The weights can be determined according to various methods (see Nijkamp et al., 1990; Eckenrode, 1965) for an overview of these methods). PROMETHEE does not provide specific guidelines for determining these weights, but assumes that the decision-maker is able to weigh the criteria appropriately, at least when the number of criteria is not too large.

2.1.3. The preference function

The preference function (P_j) translates the difference between the evaluations (i.e., scores) obtained by two alternatives (a and b) in terms of a particular criterion, into a preference degree ranging from 0 to 1.

Let

$$P_j(a,b) = G_j[f_j(a) - f_j(b)], \tag{1}$$

$$0 \leqslant P_j(a,b) \leqslant 1, \tag{2}$$

be the preference function associated to the criterion, $f_j(\cdot)$ where G_j is a nondecreasing function of the observed deviation (d) between $f_j(a)$ and $f_j(b)$.

In order to facilitate the selection of a specific preference function, six basic types have been proposed (Brans, 1982; Brans and Vincke, 1985; Brans and Mareschal, 1994).

2.1.4. Individual stakeholder group analysis

PROMETHEE permits the computation of the following quantities for each stakeholder r ($r = 1, \ldots, R$) and alternatives a and b:

$$\begin{cases} \pi_r(a,b) = \sum_{j=1}^{k} P_j(a,b) w_{r,j}, \\ \phi_r^+(a) = \sum_{x \in A} \pi_r(x,a), \\ \phi_r^-(a) = \sum_{x \in A} \pi_r(a,x), \\ \phi_r(a) = \phi_r^+(a) - \phi_r^-(a). \end{cases} \tag{3}$$

For each alternative a, belonging to the set A of alternatives, $\pi(a,b)$ is an overall preference index of a over b, taking into account all the criteria, $\phi_r^+(a)$ and $\phi_r^-(a)$. These measure respectively the strength and the weakness of a vis-à-vis the other alternatives. $\phi_r(a)$ represents a value function, whereby a higher value reflects a higher attractiveness of alternative a. We call $\phi_r(a)$ the net flow of alternative a for stakeholder k.

For each stakeholder the three main PROMETHEE tools can be used to analyse the evaluation problem:

- the PROMETHEE I partial ranking;
- the PROMETHEE II complete ranking;
- the GAIA plane.

The PROMETHEE I partial ranking provides a ranking of alternatives. In some cases, this ranking may be incomplete. This means that some alternatives cannot be compared and, therefore, cannot be included in a complete ranking. This occurs when the first alternative obtains high scores on particular criteria for which the second alternative obtains low scores and the opposite occurs for other criteria. The use of PROMETHEE I then suggests that the decision-maker should engage in additional evaluation efforts.

PROMETHEE II provides a complete ranking of the alternatives from the best to the worst one. Here, the net flow is used to rank the alternatives.

The geometrical analysis for interactive aid (GAIA) plane displays graphically the relative position of the alternatives in terms of contributions to the various criteria (Fig. 1).

Additional tools such as the "walking weights" and the decision axis can be used to further analyse the sensitivity of the results in function of weight changes (cf. Brans and Mareschal, 1994).

2.1.5. Overall, multi-stakeholder analysis

In the phase of the overall, multi-stakeholder analysis, the points of view of the different actors are pooled (see Macharis et al., 1998). The same PROMETHEE tools as used in the individual stakeholder analysis are available to the decision-maker. In the GAIA plane, any conflicts among the various stakeholders' points of view can be visualised. This can provide useful insights into the trade-offs that will need to be made among the various stakeholders' interests.

The global net flow ϕ_G is calculated as a weighted average of the individual net flows:

$$\phi_G(a_i) = \sum_{r=1}^{R} \sum_{j=1}^{k} \phi_{r,j}(a_i) w_j \omega_r, \quad i = 1, 2, \ldots, n, \tag{4}$$

where ω_r represents the relative importance of stakeholder r.

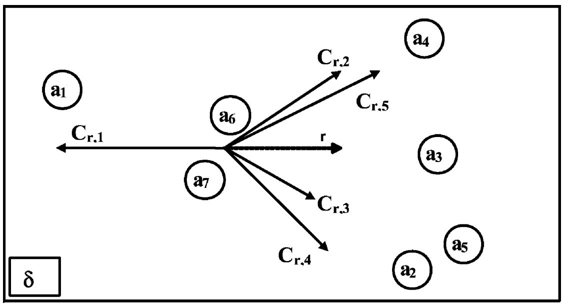

Fig. 1. The GAIA plane.

2.2. The analytical hierarchy process method

The analytical hierarchy process (AHP) (Saaty, 1982, 1988, 1995) is probably the best known and most widely used MCA approach. It is based on three principles, namely: (1) construction of a hierarchy, (2) priority setting and (3) logical consistency.

2.2.1. Construction of the hierarchy

A decision problem, centered around measuring contributions to an overall objective or focus, is structured and decomposed into its constituent parts (sub-objectives, attributes, criteria, alternatives, etc.), using a hierarchy.

2.2.2. Priority setting

The relative "priority" given to each element in the hierarchy is determined by comparing pairwise the contribution of each element at a lower level in terms of the criteria (or elements) with which a causal relationship exists. The decision maker uses a pairwise comparison mechanism, as shown in Table 1 and using a 1–9 scale.

Formally, the relative priorities (or "weight") are given by the right eigenvector (W) corresponding to the highest eigenvalue (λ_{max}), as shown in (5). The pairwise comparison matrix (Table 1) is represented by the letter A. Its standard element is $P_c(a_i, a_l)$, i.e., the intensity of the preference (in terms of contribution to a specific

Table 1
Pairwise comparison of elements in the AHP

C	a_1	...	a_l	...	a_n
a_1	1				
$\vdots$		[1]			
a_i			$Pc(a_i, a_l)$		
$\vdots$				[1]	
a_n					1

Table 2
Random consistency indices (CI*'s) (Saaty, 1988, p. 21)

r	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
CI*	0.00	0.00	0.58	0.90	1.12	1.24	1.32	1.41	1.45	1.49	1.51	1.48	1.56	1.57	1.59

criterion C) of the row element (a_i) over the column element (a_l).

$$A \cdot W = \lambda_{\max} \cdot W. \quad (5)$$

In case the pairwise comparisons are completely consistent, the matrix A has rank 1 and $\lambda_{\max} = n$. In that case, weights can be obtained by normalizing any of the rows or columns of A.

The procedure described above is repeated for all subsystems in the hierarchy. In order to synthesize the various priority vectors, these vectors are weighted with the global priorities of the parent criteria and synthesized. This process starts at the top of the hierarchy. As a result, the overall relative priority to be given to the lowest level elements (i.e., the alternatives) is obtained. These overall, relative priorities indicate the degree to which the alternatives contribute to the focus. These priorities represent a synthesis of the local priorities (i.e., the priorities within each subsystem), and reflect an evaluation process that permits to integrate the perspectives of the various stakeholders involved.

2.2.3. Consistency check

In each pairwise comparison matrix (see Table 1), a number of comparisons are redundant. In fact, in case of complete consistency, relation (6) holds:

$$P_c(a_i, a_j) = P_c(a_i, a_k)P_c(a_k, a_j) \quad \forall i, j, k. \quad (6)$$

When the pairwise comparison matrices are completely consistent, the priority (or weight) vector is given by the right eigenvector (W) corresponding with the highest eigenvalue ($\lambda_{\max}$). In that case, the latter is equal to the number of elements compared (n) (Saaty, 1988, pp. 49–50; 1986, pp. 847ff; 1995, pp. 78–79). In case the inconsistency of the pairwise comparison matrices is limited, slightly ($\lambda_{\max}$) deviates from n. This deviation ($\lambda_{\max} - n$) is used as a measure for inconsistency. This measure is divided by $n - 1$. This yields the average of the other eigenvectors (Forman, 1998, p. 301). Hence, the "consistency index" (CI), is given by (7).

$$\mathrm{CI} = \frac{\lambda_{\max} - n}{n - 1}. \quad (7)$$

The final consistency ratio (CR), on the basis of which one can conclude whether the evaluations are sufficiently consistent, is calculated as the ratio of the consistency index (CI) and the random consistency index (CI*), as indicated in (8). The random consistency indices (the CI*'s given in Table 2) correspond to the degree of consistency that automatically arises when completing at random reciprocal matrices (as shown in Table 1) with the values on the 1–9 scale.

$$\mathrm{CR} = \frac{\mathrm{CI}}{\mathrm{CI}^*}. \quad (8)$$

Saaty (1982, pp. 82–83) has argued that the inconsistency should not be higher than 10% ($\mathrm{CR} \leqslant 0.10$). An inconsistency level higher than 10% means that the consistency of the pairwise comparisons is insufficient. [1]

The CR for the whole hierarchy (CRH) is determined on the basis of the CTs and CI*'s for each pairwise comparison matrix. This procedure is explained in detail in Saaty (1988, pp. 83–84).

Recently a few authors have presented strong objections against the meaning of this consistency index (Bana e Costa and Vansnick, 2000).

3. A comparative analysis of PROMETHEE and AHP

Both PROMETHEE and AHP have strengths and weaknesses. This section includes a brief

[1] In some applications (namely, the computer program Expert Choice™ developed by Forman, 1998). this ratio is called the inconsistency ratio (ICR), because it actually provides a measure of inconsistency rather than consistency.

comparative analysis of the following elements in the two MCA methods: the underlying value judgments, the structuring of the problem, the treatment of inconsistencies, the determination of weights, the evaluation elicitation, the management of the rank reversal problem, the support of group decisions, the availability of software packages and the possibility to visualise the problem.

3.1. Underlying value judgments

The AHP-method can be considered as a complete aggregation method of the additive type (Kamenetzky, 1982, p. 712; Roy and Bouyssou, 1993, p. 202). The problem with such aggregation is that compensation (i.e., trade-offs) between good scores on some criteria and bad scores on other criteria can occur. Detailed, and often important, information can be lost by such aggregation. This is not the case with the PROMETHEE-I method, because such trade-offs are avoided. The dominance relation is enriched rather than impoverished. With the PROMETHEE-II method, the partial ranking of PROMETHEE-I is forced into a complete ranking of the alternatives; this may also lead to the loss of data.

3.2. The structuring of the problem

The AHP method has the distinct advantage that it decomposes a decision problem into its constituent parts and builds hierarchies of criteria. By doing so, the decision problem is unbundled into its smallest elements. Here, the importance of each element (criterion) becomes clear. PROMETHEE does not provide this structuring possibility. In the case of many criteria (more than seven), it may become very difficult for the decision maker to obtain a clear view of the problem and to evaluate the results.

3.3. Treatment of inconsistencies

The acceptance of limited inconsistency and the possibility of managing this issue is often considered as an advantage of the AHP method (Harker and Vargas, 1987). PROMETHEE permits, through sensitivity analysis, to establish the highest allowable deviations from the original weights (Brans, 1996), before the ranking of the alternatives is altered.

3.4. Determination of the weights

In order to determine the weights in the AHP, the following question needs to be answered: "How much more is option A contributing to a higher goal than option B?" On the basis of a sequence of such pairwise comparisons, the relative priorities (weights) are determined, using the eigenvector method. The weights should be seen as "the relative contribution of an average score (averaged over all options taken into account) of the elements (of a lower level) to each criterion (of a higher level)". The interpretation of these weights is apparently not trivial (Belton, 1986). If the questions asked to obtain the weights are not properly formulated, anomalies can occur. With PROMETHEE, no specific guidelines are provided to determine the weights. In addition, the generalised criteria need to be defined, which may be difficult to achieve by an inexperienced user.

3.5. The evaluation elicitation

With AHP, the decision problem is decomposed into a number of subsystems, within which and between which a substantial number of pairwise comparisons need to be completed. This approach has the disadvantage that the number of pairwise comparisons to be made, may become very large (more specifically: $n(n-1)/2$). PROMETHEE needs much less inputs. Only the evaluations have to be performed of each alternative on each criterion.

Furthermore, an important disadvantage of the AHP method is the artificial limitation of the use of the 9-point scale. For instance, if alternative A is five times more important than alternative B, which in turn is five times more important than alternative C, a serious evaluation problem arises. The AHP method cannot cope with the fact that alternative A is 25 times more important than alternative C (see also Murphy, 1993; Belton and Gear, 1983; Belton, 1986). According to Saaty (1982, pp. 28–29) the human brain is not able to compare stimuli which differ too much in size. One should in such cases create hierarchically arranged clusters with elements that are comparable when using a nine-point scale. In practice, this implies that artificially created subclasses may be introduced.

3.6. The rank reversal problem

Both methods suffer from the rank reversal problem. This means that, in some cases, the ranking of the alternatives can be reversed when a new alternative is introduced (this problem was first identified, for the AHP method, by Barzilai et al. (1987), Belton and Gear (1983, 1985), Dyer (1990) and Holder (1990) and for PROMETHEE by De Keyser and Peeters (1996)). In the AHP, rank reversal is likely to occur e.g., when a copy or a near copy of an existing alternative is added to the set of alternatives that are being evaluated. In case one has to make a choice between mutually exclusive alternatives, one should make the pairwise comparisons according to the ideal mode, thereby avoiding rank reversal problems. In the other cases, rank reversal may be legitimate (see Section 2.2).

3.7. Group decisions

Both methods foresee support to aid group-level decision-making through consensus. In the AHP method, this is done by calculating the geometric mean of the individual pairwise comparisons (Zahir, 1999a). In PROMETHEE, the overall decision can be made after calculating the weighted sum of the individual net flows.

3.8. Software packages

The two corresponding software packages, namely DECISION LAB (for PROMETHEE) and EXPERT CHOICE (for AHP) are both very user-friendly.

3.9. Visualisation of the problem

The final goal of an MCA, namely to foster discussion on a problem, requires a good visual projection of the problem (Pomerol and Barba-Romero, 1993). By Zahir (1999a,b). The visualisation of the decision problem is better when using PROMETHEE. The GAIA plane is a powerful tool to identify conflicts between criteria and to group the alternatives. However, a corresponding visualisation has also been developed for the AHP method.

When new projects or alternatives are included in DECISION LAB (the software program for PROMETHEE), this can be done without many changes. The criteria and generalised criteria may remain the same. Only the evaluations of the new projects in terms of scores on the existing criteria have to be introduced into the model. As regards the AHP method, however, all the pairwise comparisons at the lowest level in the hierarchy (i.e. at the level of the alternatives) need to be re-entered into the computer program EXPERT CHOICE.

The advantages and disadvantages of both MCA methods are summarized in Table 3.

The underlying paradigm of the AHP method allows for important concerns of individual stakeholder groups, and the related unsatisfactory scores, to be compensated by positive scores on other criteria and for other stakeholder groups. This type of aggregation can lead to the loss of important information. The outranking methods permit to retain more information by starting from the premise that some alternatives may not be comparable. The PROMETHEE-I ranking allows for a partial ranking. However, the underlying paradigm of PROMETHEE is also labelled as a disadvantage in Table 3 above because PROMETHEE-II aggregates the values so as to obtain a complete ranking.

All MCA methods are obviously faced with this problem, because they build on the premise that various criteria need to be taken into account simultaneously, in order to generate an overall evaluation of a number of alternatives. As regards the determination of the weights, a negative evaluation was given to both methods. With AHP, weights are obtained through a sequence of pairwise comparisons, but the adoption of this method may be hindered by the limited 1–9 scale of Saaty. With PROMETHEE, no guidelines are provided to determine weights.

Table 3
Summary comparison table between the AHP- and the PROMETHEE-method

	AHP	PROMETHEE
Paradigm of the underlying method	−−	−
Structuring the problem	++	+
The treatment of inconsistencies of the DM	Ex-ante	Ex-post
Determination of the weights	+	−
Amount of evaluations to give	−	+
9-point scale	−	+
Rank reversal problem	−	−
Software	++	++
Visualisation	+	++
Flexibility of the software package	−−	+

The scores are based on the argumentation of before and are representing the following values: (−−) the approach that was followed has important disadvantages; (−) the approach that was followed has disadvantages; (+) the approach that was followed has advantages; (++) the approach that was follwed has important advantages.

Table 3 does suggest that a number of favourable characteristics of the AHP method could enhance PROMETHEE, namely at the level of the structuring of the decision problem and the determination of weights.

4. Recommendations for enhancing PROMETHEE through introducing AHP features: Creating operational synergies

4.1. Hierarchy

The use of a tree structure to decompose the decision problem constitutes an important advantage of the AHP-method. This allows the decision-maker to gain a better overview of the decision problem because it is unbundled into its constituent parts (sub-objectives), which in turn can again be subdivided into smaller elements.

With PROMETHEE, we are faced with sometimes very large evaluation tables that can include more than seven alternatives and more than seven criteria. At that point, PROMETHEE becomes little more than a black box to assist in solving decision-making problems. In this context, it could be viewed as an enhancement of PROMETHEE if a tree-structure were adopted.

The extension of PROMETHEE to a group environment, using the DECISION LAB software already provides the option to build three levels into a model. The first level permits the analysis at the level of various individual decision-makers. These various perspectives can then be pooled in the level above. A third (separate) level then allows the actions (or alternatives) themselves to be pooled together in a specific group and can be further treated as a single action group. The pooling at a higher level is based on calculating the weighted sum of the underlying net flows (Macharis et al., 1998). If this idea were extended further, it could lead to the development of a hierarchical tree structure within PROMETHEE.

4.2. Determination of the weights

With PROMETHEE, a weight $w_j = 1$ may be chosen for criterion j. For each other criterion, the weight is then measured relative to the jth criterion. These weights can be determined by asking the question: "How much is criterion i more important than criterion j (with respect to the higher goal)?"

When completing the AHP matrix, at the jth row the question becomes: "How much more important is criterion j than criterion i (with respect to the higher goal)?"

There are, however, a few restrictions.

- It is a necessary condition that the pairwise comparison matrix be consistent.
- One has to give up the often criticized 9-point scale.

The following comments can be formulated regarding these restrictions. With PROMETHEE, we have a single set of weights w_j, $j = 1, \ldots, n$, with n as the number of criteria, while in the pairwise comparison matrix we have n columns, which would lead to n sets of n weights. If the pairwise comparison matrix is consistent, we end up with a matrix of rank 1. Hence, there is a single linear independent row, which, thanks to the reciprocal character of the pairwise comparison matrix, is equivalent to a single linear independent column. This permits to define a single set of weights. In general terms, any column can be taken to determine the PROMETHEE weights, because the choice $w_j = 1$ is arbitrary with respect to the index j, given the relative measure of the weights in PROMETHEE.

Example. Consider the following pairwise comparison matrix:

D	C_1	C_2	C_3
C_1	1	3	6
C_2	1/3	1	2
C_3	1/6	1/2	1

In this matrix, three criteria (C) are compared in pairs in terms of their relative importance to the decision-maker (D). It is easy to check that this consistent matrix has rank 1, since the second and third row are the equivalent of the first row divided by respectively the second and third element of the first row.

If we take, e.g., the second column (3, 1, $\frac{1}{2}$) as the set of weights in PROMETHEE, this generates the following basic observations:

- C_1 is moderately more important in the decision process than C_2;
- C_3 is slightly less important in the decision process as C_2.

These statements are identical to the ones generated by the second column of the pairwise comparison matrix, if read from an AHP-perspective.

The second restriction is a consequence of the first one. If we want to guarantee consistency of the matrix, we have to abandon the 9-point scale.

If the above is accepted, the problem is reduced to obtaining a consistent matrix from the decision-maker. In the following, we present an interactive procedure to obtain such a matrix.

4.2.1. Procedure for the determination of a consistent matrix

First a pairwise comparison is performed in the standard AHP-manner in order to determine the first row of the matrix.

Since we want the matrix to be consistent and it is a reciprocal matrix, we can generate from this first row all the remaining rows so as to obtain the complete matrix.

The second interactive step consists in presenting this matrix to the decision-maker, who may change his/her mind with respect to several "implicit" pairwise comparisons (those automatically generated from the first column). Here, two possibilities arise.

(1) The decision-maker agrees with the matrix, which implies that the procedure is completed.
(2) The decision-maker wants to change one or more elements of the matrix. Let a_{ij} ($j > i$) be this element. Since we want the final matrix to be a' consistent reciprocal matrix, the following equation holds:

$$a_{ij} = \prod_{i \leqslant k \leqslant j-1} a_{k,k+1}. \tag{9}$$

Hence, the decision-maker will need to alter some fundamental elements such that their product equals the value he gave to the specific element a_{ij}.

It should be remembered that every change the decision-maker makes, will affect the matrix. Hence, the consequences of every change must be shown to him, i.e. all the elements that will change.

The above procedure may lead to a sequence of changes. Therefore, we propose to ask the decision-maker to indicate all the elements he/she would like to change. The element with the largest column index j^* determines the row ($j^* - 1$) where the questioning will start on changes due to relation (9).

Table 4
Scores of the pairwise comparison

D	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	1	2	3	2	5	2/3	7	3

Table 5
Consistent matrix generated from Table 4

D	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	1	2	3	2	5	2/3	7	3
C_2	1/2	1	3/2	1	5/2	1/3	7/2	3/2
C_3	1/3	2/3	1	2/3	5/3	2/9	7/3	1
C_4	1/2	1	3/2	1	5/2	1/3	7/2	3/2
C_5	1/5	2/5	3/5	2/5	1	2/15	7/5	3/5
C_6	3/2	3	9/2	3	15/2	1	21/2	9/2
C_7	1/7	2/7	3/7	2/7	5/7	2/21	1	3/7
C_8	1/3	2/3	1	2/3	5/3	2/9	7/3	1

While considering a certain row, no changes are applied to the lower rows (except those to maintain consistency), which implies that a change in value of a single element determines the values of all the other elements of this row. Once a decision is made on this row, we move to a higher row in the matrix and perform any necessary changes, etc.

This last part could also lead to an infinite number of iterations, but it has the advantage that the decision-maker gets better guidance as compared to the case of ad random changes of elements. The latter approach may lead to very complex changes in the matrix that are not always understandable to the decision-makers.

Example. Let us consider an example of the proposed procedure.

Step 1. Consultations with the decision-maker lead to the row of scores through pairwise comparison presented in Table 4.
From this row we are now able to generate the consistent matrix presented in Table 5.

Step 2. Assume that the decision-maker wants to change the underlined elements in this matrix, i.e. the elements $\{a_{24}, a_{15}, a_{26}, a_{57}\}$.

We start to question the decision-maker on the elements of the sixth (i.e. $j^* - 1$) row, warning him/her what the consequences will be for the elements of the seventh and eighth column if we change one of these elements. Suppose that the decision-maker wants the following value for element a_{57}: 2. Without changing the elements of the sixth row in the upper triangle, we obtain the matrix presented in Table 6.

Table 6
Matrix with value 2 for element a_{57}

D	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	1	2	3	2	5	**20/21**	**10**	**30/7**
C_2	1/2	1	3/2	1	5/2	**10/21**	**5**	**15/7**
C_3	1/3	2/3	1	2/3	5/3	**20/63**	**10/3**	**10/7**
C_4	1/2	1	3/2	1	5/2	**10/21**	**5**	**15/7**
C_5	1/5	2/5	3/5	2/5	1	**4/21**	**2**	**6/7**
C_6	**21/20**	**21/10**	**63/20**	**21/20**	**21/4**	1	21/2	9/2
C_7	**1/10**	**1/5**	**3/10**	**1/5**	**1/2**	2/21	1	3/7
C_8	**7/30**	**7/15**	**7/10**	**7/15**	**7/6**	2/9	7/3	1

Table 7
Matrix with value 2 for element a_{45}

D	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8
C_1	1	2	3	2	**4**	**16/21**	**8**	**24/7**
C_2	1/2	1	3/2	1	**2**	**8/21**	**4**	**12/7**
C_3	1/3	2/3	1	2/3	**4/3**	**16/63**	**8/3**	**8/7**
C_4	1/2	1	3/2	1	**2**	**8/21**	**4**	**12/7**
C_5	**1/4**	**1/2**	**3/4**	**1/2**	1	4/21	2	6/7
C_6	**21/16**	**21/8**	**63/16**	**21/8**	21/4	1	21/2	9/2
C_7	**1/8**	**1/4**	**3/8**	**1/4**	1/2	2/21	1	3/7
C_8	**7/24**	**7/12**	**7/8**	**7/12**	7/6	2/9	7/3	1

In the matrix in Table 6, all the changed elements are represented in bold. The changes made by the decision-maker in the fifth row have implications on elements, which the decision maker initially did not explicitly intend to change. For example, if the decision maker wants to change element a_{45} (e.g., $a_{45} = 2$), then the fourth row will need to be corrected, which would lead to the matrix: presented in Table 7.

This part of the procedure is repeated until the first row has been modified (if still necessary).

The complete matrix is then considered again by the decision-maker and, if necessary, the second step is repeated until he/she is satisfied with the matrix.

5. Conclusions

This paper has argued that operational synergies can be achieved by integrating into PROMETHEE, a number of elements usually associated with AHP. More specifically, PROMETHEE can be improved by introducing a tree-like structure similar to the one found in AHP, in order to unbundle the decision-making problem into subcomponents. In addition, a new weighing approach was suggested that could greatly benefit PROMETHEE. Until now, this MCA tool did not provide any formal guidelines for weighing. The advantage of the weighing system proposed in this paper, as compared with the suggestion of Hens et al. (1992) to simply replicate the AHP weighing; approach in a PROMETHEE context, is that the decision maker needs to perform only a limited number of pairwise assessments. More precisely, these assessments are required only for the first row (i.e., the non-redundant comparisons). The other pairwise comparisons are generated automatically after the formal assessment for the first row. The decision-maker obviously needs to perform a final check of the overall impacts of his choices on the entire evaluation matrix, but the systematic, active assessment of all pairs of elements, even the redundant ones, as is characteristic for AHP, can be avoided. Finally, the new approach also avoids the use of the 1–9 Saaty evaluation scale. Although this scale can be viewed as consistent with the bounded rationality restrictions facing human decision makers, the approach adopted here permits more flexibility. In more general terms, this paper suggests that future academic research should focus on comparative assessments of the relative strengths and weaknesses of alternative MCA approaches. By combining the strengths of various approaches into a single MCA tool, operational synergies can be achieved that can greatly benefit decision makers.

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