Discrete Optimization

A network-consistent time-dependent travel time layer for routing optimization problems

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In the vehicle routing literature, there is an increasing focus on time-dependent routing problems, where the time (or cost) to travel between any pair of nodes (customers, depots) depends on the departure time. The aim of such algorithms is to be able to take recurring congestion into account when planning logistics operations. To test algorithms for time-dependent routing problems, time-dependent problem data is necessary. This data usually comes in the form of three-dimensional travel time matrices that add the departure time as an extra dimension. However, most currently available time-dependent travel time matrices are not network-consistent, i.e., the travel times are not correlated both in time and space. This stands in contrast to the behavior of real life congestion, which generally follows a specific pattern, appearing in specific areas and then affecting all travel times to and from those areas. As a result of the lack of available network-consistent travel time matrices, it is difficult to develop algorithms that are able to take this special structure of the travel time data into account.

In this paper, we present a method to generate time-dependent travel time matrices that are both spatially and temporally correlated and hence network-consistent. The novel travel time generation model is conceptually elegant, requires a minimal amount of input data and provides an efficient and effective way of storing the time-dependent travel time matrices. In this way, the data sets generated by our approach achieve a much larger degree of realism, without suffering from a decreased usability. The characteristics and applicability of the newly presented model are demonstrated on a time-dependent vehicle routing problem. The method to generate network-consistent time-dependent travel time data has been implemented in Java and is available for download (http://antor.ua.ac.be/network-consistent).

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1. Introduction

There is very little doubt that road congestion levels all over the world are increasing rapidly (see, e.g., Downs, 2004), causing enormous economic opportunity costs, as well as placing a heavy burden on the environment. In general, congestion levels vary during the day due to variations in the demand for road transport, causing a peak-hour congestion surge every morning and evening. As a result, travel times between any two points in the road network are time-dependent, i.e., they depend on the departure time.

The road network is not uniformly affected by congestion: traffic jams occur mostly in areas where the road network is dense or the population is large, and is therefore most notorious in metropolitan areas. Traffic jams are usually not confined to single roads, but spread throughout the network and affect roads that are located in the vicinity of the location where congestion first occurred (for which reason congestion is often modelled as a fluid, see e.g., Berthelin et al., 2008). Congestion is therefore both temporally and spatially correlated, a property which we call network-consistent.

To better explain the concept of network-consistent travel times, consider the following situation. Suppose A, A’, B and B’ are customers in a vehicle routing problem. Customer A’ is located in the close vicinity of customer A and B’ is located close to B. To be network-consistent, the travel time profile (i.e., the travel time as a function of the departure time) between customers A and B should closely resemble that between customers A’ and B’. Clearly, network-consistency can be achieved by increasing and decreasing travel times between all pairs of customers simultaneously. Network-consistency is therefore a necessary, but not a sufficient condition to generate realistic time-dependent vehicle routing instances. It is clear, however, that any method that randomly changes travel times between pairs of customers will not achieve network-consistency.

A graphical representation of the spatio-temporal correlation required by network-consistency is provided in Fig. 1, where the geographical region around C is congested. Consequently, roads A–C and B–C are both affected by the same congestion zone (Fig. 1a). In the travel time profiles, i.e., the graph that depicts
the travel time between two nodes as a function of the departure time, travel delays appear on both \( A-C \) and \( B-C \). Since the (time-dependent) travel time is given at the starting time of the vehicle at the origin, the travel time on \( A-C \) (Fig. 1b) increases sooner than the travel time on \( B-C \) (Fig. 1c). The reason for this is that the travel time to reach the congestion area from \( A \) is longer than from \( B \). A vehicle starting from \( A \) that arrives in \( C \) when congestion is present will have started sooner than a vehicle starting from \( B \) and arriving at the same time in \( C \).

In the routing literature, the node network layer is usually represented by a complete directed graph \( G = (V, E) \), where \( V = \{0, 1, \ldots, n\} \) is a set of vertices representing the depot \( 0 \) and the customers \( \{1, \ldots, n\} \), and \( E = \{(i,j)|i,j \in V\} \) the set of edges. Note that we will use the term node to signify a location that can be visited in a routing problem, i.e., either a customer or a depot. The (time-independent) travel time matrix \( TT \) contains for each edge \( ij \) the travel time \( TT_{ij} \) when traveling from node \( i \) to node \( j \). In a time-dependent routing problem, this travel time is a function of the moment in time \( t \) when the vehicle leaves the origin \( i \). This is denoted as \( TT_{ij}(t) \), i.e., the travel time needed to cover the distance to go from \( i \) to \( j \), leaving the origin \( i \) at time \( t \). The free flow travel time (i.e., the minimum time needed to travel from \( i \) to \( j \) without any delay) is written as \( TT_{ij}^{f} \).

In the vehicle routing literature, the added value of using time-dependent travel times is generally acknowledged (Malandraki et al., 1992; Ichoua et al., 2003; Donati et al., 2003). By developing a method to optimize the starting times of the vehicles in a vehicle routing problem in the presence of time-dependent travel times and time windows, Kok et al. (2011) convincingly show that ignoring time-dependent travel times is very likely to yield infeasible solutions. However, current travel time generation methods do not account for the specific congestion pattern observed in real life and generate travel time profiles that are not spatially correlated and thus unrealistic. As a corollary, a large majority of existing routing optimization algorithms, with the notable exception of the algorithms developed by Kok et al. (2012), are not designed to exploit the specific travel time pattern caused by congestion. It is reasonable to assume that algorithms that are designed to work on realistic time-dependent travel time data and that, e.g., avoid congestion-sensitive zones during certain periods of the day, will also perform better in real life.

The main contribution of this paper is an innovative method to generate time-dependent travel times that are guaranteed to be network-consistent, allowing for the development of routing algorithms that can exploit the specific congestion pattern that also exists in real life. The method described in this paper has many advantages and can be applied to any routing problem defined on a time-dependent travel time matrix.

In Section 2 the literature on time-dependent vehicle routing is reviewed, focusing not on the algorithms to solve such problems, but rather on the currently used methods to generate and represent time-dependent travel times. Our novel travel time generation method is proposed in Section 3. The implementation and applicability of the newly presented model to vehicle routing problems is discussed in Section 4. Conclusion and recommendations for future research are provided in Section 5.

### 2. Literature review

In recent years, research in vehicle routing problem has shifted its focus from the classical VRP (Laporte, 1992) with constant travel times between nodes to more advanced variants, for which travel times are not constant. In this category, routing problems with dynamic travel times (Fleischmann et al., 2003; Potvin et al., 2006; Chen et al., 2006; Van Woensel et al., 2008) and stochastic travel times (Laporte et al., 1992; Gendreau et al., 1996) can be mentioned. Such problems are more realistic, because they account for travel time variability, which is overwhelmingly present in real life and has many causes (see, e.g., NCHRP, 2003).

Recently, research has focused on vehicle routing problems with time-dependent travel times (Malandraki et al., 1992; Ichoua et al., 2003; Donati et al., 2003; Haghani and Jung, 2005; Potvin et al., 2006; Chen et al., 2006; Van Woensel et al., 2008; Lecluyse et al., 2009).

Although it is possible in principle to collect real-life time-dependent travel time data from real-life measurements (Fleischmann et al., 2004; Ando and Taniguchi, 2006), such an approach is not viable for testing vehicle routing algorithms due to its complexity on the one hand and its inflexibility (e.g., the impossibility to generate data sets of arbitrary size) on the other. Most vehicle routing algorithms are therefore tested on artificially generated travel time data.

Several approaches to generate time-dependent travel time data have been proposed. One of the first approaches defines the travel time between two nodes (customers/depots) as a function of the distance and the time of the day by using a time-dependent cost factor, resulting in a piecewise constant distribution of the travel time (Malandraki et al., 1992; Potvin et al., 2006). Although this travel time generation technique is easily applied, its ability to model realistic travel time profile is limited. Randomly modifying these travel times based on predefined binomial distributions
(Kenyon and Morton, 2003) only slightly contributes to the imitation of real-life travel variations. In these models, the FIFO principle is not necessarily satisfied (Ichoua et al., 2003), i.e., it is not guaranteed that a vehicle that leaves later at the origin than another vehicle also arrives later at the destination.

Temporal consistency is satisfied by working with step-like speed distributions. In such distributions, the time is divided into discrete periods and the travel time on an edge changes linearly during each period, in such a way that the travel time at the end of a period equals the travel time at the beginning of the next period (Ichoua et al., 2003; Donati et al., 2003). By calculating the travel speed based on the design speed of each edge and the ratio of the travel speed to the design speed at that time, Haghani and Jung (2005) are able to deal with any kind of travel time variation. Although travel time generation based on speed transformations is easily implemented, spatial correlation is not accounted for.

Travel time variations can also be modeled using analytical queuing models, which model the behaviour of traffic flows as a function of the most relevant physical and geographical determinants (i.e., free flow speed, maximum road capacity, travel variability due to the weather, etc.) (Van Woensel and Vandaeye, 2006; Van Woensel et al., 2008; Lecluyse et al., 2009). As this method relies on vehicle counts, it is difficult to apply to theoretical routing models while satisfying network-consistency.

To the best of our knowledge, the only other existing method to generate network-consistent time-dependent travel times is due to Kok et al. (2012). Contrary to the method outlined in this paper, the method of Kok et al. (2012) generates network-consistent travel times by explicitly considering the underlying road network. The authors define a speed profile for every arc in the road network and then generate the (time-dependent) shortest paths between customers in the network in 15 minute intervals, either interpolating or re-calculating the length of a time-dependent shortest path if the starting time of the vehicle does not correspond to the starting time of a 15 minute interval. The method proposed in this paper differs substantially from the one proposed in Kok et al. (2012), most notably because it operates directly on inter-customer distances and does not rely on the knowledge of an underlying road network. For most vehicle routing instances, only the inter-customer distances are given and our method, unlike the one by Kok et al. (2012), can therefore be used to transform all time-independent routing problem instances to time-dependent ones, even if the underlying road network is not known. For the same reason, our method requires considerably less input data. Secondly, our method does not require an artificial discretization of the departure time into (15 minute or other) time buckets, and stores all travel times between any pair of customers as a piece-wise linear function of the departure time. Also, it does not require any re-solving of a series of time-dependent shortest path problems to exactly calculate the total travel time of a given vehicle routing solution. Besides changing the order in which customers are visited, and the assignment of customers to vehicles, congestion can also be avoided by taking a different route between a given pair of customers. Our method implicitly assumes that the time-dependent travel time for every possible departure time uses the shortest path available at that time. The method of Kok et al. (2012) explicitly calculates these time-dependent shortest paths, and may therefore be somewhat more realistic.

An important decision concerns the way to store the time-dependent travel time data. A first approach is to divide the planning period in fixed-width time buckets of, e.g., 1 hour (see Fig. 2a). This approach is used e.g., Donati et al., 2003; Haghani and Jung, 2005; Van Woensel et al., 2008; Lecluyse et al., 2009). Naturally, there is a trade off between the accuracy of the travel times and the required storage capacity, both of which increase with the number of time buckets. Travel time retrieval is very fast as the starting time at the origin of an edge uniquely identifies the travel time record in the travel time matrix. However, this method cannot guarantee to keep track of all relevant fluctuations in the travel time profile. On top of that, in case of a long period of time with (approximately) the same travel times, this method stores data inefficiently.

A second set of models found in the literature account more explicitly for rush-hour periods by adjusting the bucket width to the expected travel time profile (Ichoua et al., 2003; Potvin et al., 2006), still using the same bucket boundaries for all edges (see Fig. 2b). Although such travel time profiles account for travel time fluctuations due to rush-hours, these rush-hours do not necessarily occur at the same departure time for all edges. Additionally, not every edge should have the same number of time zones associated with a different travel time trend. As a result, it is not trivial to implement realistic travel scenarios using this method. The grouping of the edges is also prone to violating the spatial correlation between travel times.

A final method is to divide the travel time into time buckets that may be different for each edge (see Fig. 2c). The main advantage is that this data representation method guarantees to keep track of all travel time fluctuations on all edges, without redundancy in the number of travel time records. The problem is that these travel time matrices have to be created artificially in theoretical vehicle routing problems, which makes it hard to control for the network-consistency. In the literature, Malandraki et al. (1992), Chen et al. (2006) arbitrarily create variable time zones for all edges, without mentioning a procedure to control for spatial consistency.

3. A new network-consistent time-dependent travel time layer

In this section, we describe the network-consistent time-dependent travel time layer. A list of the symbols introduced in this section, can be found in Appendix A. The new layer can be superimposed on an existing graph of any routing problem, provided that (a) node coordinates and (b) free-flow travel times between any pair of nodes are given. Free flow travel times between a pair of nodes can be (proportional to) the Euclidean distance between the nodes, but this is not necessary. To generate the time-dependent travel time matrix, the directed graph \( G \) is projected on a Euclidean plane, using the coordinates of every node \((x_i, y_i)\). For each edge in the graph \((i, j) \in E\) a straight line segment connecting points \((x_i, y_i)\) and \((x_j, y_j)\) is drawn. We call such a line segment a link and define the length \(d_{ij}\) as the Euclidean distance between \(i\) and \(j\). The link is used to determine the amount of congestion and thus the delay on an edge at each point in time. The length of the link \(d_{ij}\) does not necessarily have any relationship with the free-flow travel time \(T_{ij}\) on the edge corresponding to it. To avoid overly wordy phrasings, we will often use the term link to signify both the line segment between the Euclidean projection of two nodes and the corresponding edge in the graph of the routing problem.

3.1. Generating travel times through congestion circles

Temporal and spatial congestion effects are captured through the use of congestion circles. Each congestion circle \(c_k\) (indicating the kth congestion circle) represents a congested area and has a fixed circle center and a time-dependent radius. The travel delay on an edge is determined by the intersection with the congestion circle\(s\) of the link corresponding to that edge. Since the radius of the congestion circles changes over time, so does the travel delay.

The effect of a congestion circle on the travel time on an edge is determined using Eq. (1). The travel time of a vehicle moving from \(i\) to \(j\) on link \(ij\) is determined by the length of the link section within
one or more congestion circles during its travel. The travel delay on an edge is proportional to the ratio of the length of the link section inside the congestion circle to the link length $d_{ij}$. This is illustrated in Fig. 3 for the effect of a single congestion circle $C_k$ on a link $ij$. For conciseness reasons, we abbreviate the index $C_kij$, that represents the effect of congestion circle $C_k$ on link $ij$ as $c$. It is assumed that the travel speed on a link, both within and outside of a congestion circle, is constant. The fact that a vehicle that travels on an edge affected by a congestion circle, only reaches the edge of the congestion circle after some time, is also taken into account. Generally therefore, the time-dependent travel time for a vehicle (which is expressed as a function of the departure time of the vehicle) increases earlier than the point in time at which the congestion circle first intersects with the link. The difference between the point in time at which the travel time for the vehicle starts increasing and the point in time at which the congestion circle first intersects

![Travel time representation possibilities](image)
the minimum link travel time observed in, e.g., Fig. 4. This effect is explained in detail later in this paper and can be clearly observed in, e.g., Fig. 4.

The section of the link inside the congestion circle is called the congested link section (CLS). The calculated travel delay is then added to the minimum link travel time $TT_{ij}$. If the link lies entirely within the congested area, the delay is maximal. As the travel delay due to congestion might differ between links (some links may be affected worse than others), the travel delay is multiplied by a link-specific parameter $\delta_{ij}$. To enhance readability of the equations, we define $\gamma = c_{ij}$ to refer to the combination (i.e., the impact) of the $k$th congestion circle on the link $ij$. The length of the congested link section caused by congestion circle $k$ on link $ij$ at time $t$ is represented as $CLS_{ij}(t)$. If a link is affected by a single congestion circle, the time-dependent travel time is given by the following formula.

$$TT_{ij}(t) = TT_{ij} \left(1 + \delta_{ij} \frac{CLS_{ij}(t)}{d_{ij}}\right) \quad (1)$$

The time-dependent circle radius mimics the time-dependent profile of a real-life congestion area. Indeed, congestion suddenly appears at a certain location. The congestion area expands, causing increasing travel delay on links that are already inside the congestion area. As the congestion area expands, congestion emerges on nearby link sections. Once the congestion area reaches its maximum, the congestion conditions remain constant for a certain period of time. Eventually, the congestion area decreases and free flow conditions gradually return on all affected links.

The methodology proves to be conceptually elegant, with a minimum of required input data to construct the congestion circles and the travel time matrix. As the travel times for all links in the network are calculated based on a set of predefined congestion circles, the travel time information can be viewed as an extra layer, on top of the node location layer. This allows the creation of multiple congestion scenarios for the same set of nodes, which is easily implemented. Data storage and retrieval is efficient and effective as the generated link travel times are stored as a series of [departure time, travel time] pairs whenever the slope of the (piecewise linear) link travel time profile changes, capturing all relevant travel time fluctuations without redundancy. By modeling the congestion in areas through the adoption of congestion circles, we ensure that congestion effects are temporally and spatially correlated.

Determining the effects of the congestion circle(s) on the link travel times requires careful calculation of the length of the congested link section for each pair of nodes and at each point in time. This has proven to be a non-trivial task, especially because (1) a congestion circle can interact with a link in several different ways and (2) different congestion circles can interact with each other on the same link. However, one of the most important advantages of the method described in this paper is that all these calculations can be performed off-line and the result can be stored in a time-dependent data set that requires only minimal storage space. The remainder of this section is devoted to explaining the basic principles underlying the calculation of the length of the congested link section (and the resulting time-dependent travel time), leaving most of the details to be found in Appendix B, C, D, E.

3.2. Impact of one congestion circle on the link travel time: base case

In this section, the impact of a single congestion circle on the link travel time profile is covered in detail for a base case, i.e., when the congestion circle at its maximum radius intersects the link twice, i.e., both intersection points lie between the end points of the link (see Fig. 4). In Section 3.3, guidelines are provided to determine the travel delay in all other cases.

As congestion is modeled using congestion circles, the analysis starts from the profile of the congestion circles (see Fig. 4a and b). The figures are aligned in such a way that departure times correspond in all three figures. First, every congestion circle $c_{ij}$ emerges at a particular location with coordinates $x_{ij}$ and $y_{ij}$ and congestion onset $(con)$ begins at time $t_{ij}^{con}$. The circle radius then increases with a constant rate towards its maximum, which is reached at time $t_{ij}^{cof}$. The travel delay remains constant during this period of full congestion $(fc)$. When the circle radius starts decreasing, congestion offset $(cof)$ sets in at time $t_{ij}^{cof}$, which also occurs at a constant rate. Finally, the congestion circle completely disappears and free flow $(ff)$ conditions set in at time $t_{ij}^{ff}$.

The time-dependent travel time matrix contains a piecewise linear function of the departure time for each origin–destination pair. This function can be completely described by its pivot points, i.e., points in which the slope of the travel time profile changes. In the newly presented methodology, a key issue is to determine which links are influenced by a congestion circle, the time when they are enclosed and the level of congestion at that time. As long as a link does not intersect with a congestion circle, the link travel time will remain at its minimum value ($TT_{ij}$), even though the congestion circle radius already increases. Only when the growing congestion area intersects with the link, congestion will appear on the link, with increasing travel delays as a consequence (Fig. 4 at time $t_{ij}^{con}$). At that time, the circle radius has a magnitude of $r_{ij}^{con}$. The location where congestion sets in is determined by the perpendicular intersection point $v_{ij}$, with coordinates $x_{ij}$ and $y_{ij}$ (see Appendix B):

$$x_{ij} = \frac{x_{v_{ij}} + x_{v_{ij}}r_{ij}^{2}}{r_{ij}^{2} + 1} + \frac{y_{v_{ij}} - y_{ij}}{r_{ij}^{2} + 1} \quad \frac{y_{v_{ij}} - y_{ij}}{r_{ij}^{2} + 1}$$

$$y_{ij} = \frac{-r_{ij}^{2}x_{ij} + x_{v_{ij}} + r_{ij}^{2}y_{ij}}{r_{ij}^{2} + 1}$$

where $r_{ij}^{1} = \frac{y_{ij} - y_{ij}}{x_{ij} - x_{ij}}$ and $r_{ij}^{2} = \frac{x_{ij} - x_{ij}}{y_{ij} - y_{ij}}$.

Congestion onset $(t_{ij}^{con})$ occurs when the circle first intersects the link. Given the moment of appearance of the congestion circle
and given the fact that the congestion circle radius and the travel time delay reach their maximum at the same time (Fig. 4a–c: \( t_{fc}^{ck} = t_{fc}^{c} \)), congestion onset time can be determined using linear interpolation (see the following equation).

\[
\begin{align*}
  t_{con}^{c} &= t_{con}^{ck} + \frac{r_{p}^{c} - r_{p}^{ck}}{C_{0}^{}}
\end{align*}
\]  

(2)

Since travel times on link \( ij \) are determined based on the time the vehicle leaves \( i \), one has to account for the time it takes to cover the distance between the origin node \( i \) and the location of the congestion onset. As such, the starting time for congestion onset on link \( ij \) \( t_{con}^{ij} \) follows:

\[
\begin{align*}
  t_{con}^{ij} &= t_{con}^{ij} - TT_{i}^{pm} \\
  TT_{con}^{ij} &= TT_{ij}
\end{align*}
\]  

(3)

where \( TT_{i}^{pm} \) represents the free flow travel time from the origin \( i \) to the first intersection point (with circle radius \( r_{p}^{i} \)).

From then onwards, the level of congestion and hence the travel time on the link will further increase, until the circle radius reaches its maximum \( r_{m}^{ij} \). The length of the resulting maximum congested link section \( c_{ls}^{m} \) is given by

\[
\begin{align*}
  c_{ls}^{m} &= 2 \sqrt{\left( r_{m}^{ij} \right)^{2} - \left( r_{p}^{ij} \right)^{2}}
\end{align*}
\]  

(4)

Using Eq. (1), the maximum link travel time \( TT_{ij}^{m} \) becomes:

\[
\begin{align*}
  TT_{ij}^{m} &= TT_{ij} \left( 1 + \delta_{ij} \min \left[ \frac{2 \sqrt{\left( r_{m}^{ij} \right)^{2} - \left( r_{p}^{ij} \right)^{2}}}{d_{ij}}, 1 \right] \right)
\end{align*}
\]  

(5)

Since the link travel time is given at the time the vehicle leaves the origin, we have to account for time needed to cover the distance between the origin and the location where the maximum delay is first encountered. Let \( v_{ij}^{m} \) be the intersection point near the origin \( i \), the minimum travel time to reach this location \( TT_{ij}^{m} \) is subtracted from the time when full congestion first occurs on the link \( t_{fc}^{ij} \), yielding the starting time of full congestion on this link \( t_{fc}^{ij} \) (Eq. (6)). For the calculation of the coordinates of the intersection points, see Appendix C.
The starting time at the congestion circle first encloses the origin (e.g., \( t^{c_{ij}}_{ij} = t^{c_{ij}}_{i} \)). This situation is valid only if the traveler moves faster than the congestion area. If however the congestion area expands faster (e.g., changing weather conditions can alter traffic conditions in a large area before one can leave that area), then the congestion onset location is the intersection point between the maximum congestion area and the link, closest to the destination (\( v^{m} \)). Indeed, as the congestion area expands faster, any location closer to the origin will result in congestion that cannot be avoided until the vehicle reaches the end of the congestion area. As a result, one has to subtract the travel time to reach this intersection point closest to the destination at free flow conditions to determine the congestion onset starting time in this case \( (t^{c_{ij}}_{ij} - T^{m}_{i}) \). If the congestion retreats faster than one can travel, then free flow condition set in when the vehicle just reaches the maximum congestion area closest to the origin \( (t^{c_{ij}}_{ij} - T^{m}_{i}) \).

3. **Magnitude of maximum travel time.** The maximum travel delay occurs whenever the congestion area is at its maximum. The maximum length of the enclosed/congested link section quantifies this maximum congestion delay \( (c_{ij}^{m}) \). E.g., for any circle center in sector II, this delay is given by

\[
\begin{align*}
\text{cls}^{m}_{ij} &= d_{ij}, \\
\text{cls}^{m}_{ij} &= \sqrt{(r^{m}_{ij} - r^{f}_{ij})^2 + ((x^{m} - x^{f})^2 + (y^{m} - y^{f})^2)}
\end{align*}
\]


where \( v^{m} \) refers to the intersection point between the link \( ij \) and the kth circle at maximum radius closest to the destination \( j \).

4. **Timing of maximum travel time.** As mentioned in the first guideline, one has to account for the distance towards the first intersection between the link and the maximum congestion area. However, if the origin is enclosed in the maximum congestion area, then the maximum delay is already felt leaving the origin before the congestion area reaches its maximum value. Therefore, the time needed to cover the link section inside the maximum congestion area should be subtracted from the time the congestion area reaches its maximum. \( t^{c_{ij}}_{ij} \). This assumes however that at that time the origin is already inside the congestion area. If this is not the case, then the full congestion starting time at the origin is approximated by the congestion onset starting time \( (t^{c_{ij}}_{ij} - T^{m}_{i}) \). A similar reasoning applies for the congestion offset.

5. **Additional pivot points.** The travel delay rate during congestion onset (offset) remains constant for as long as the link is either consistently fully enclosed in the congestion area or falls consistently only partially inside the congestion area. However, when the congestion initiates somewhere on the link, but extends beyond either end point of the link, the travel delay rate changes. In those cases (sectors II and IV in Fig. 5a) the time and congestion circle radius must be recalculated at the time when either end point is just enclosed in the congestion circle. E.g., \( v^{f} \) denotes the location on the link \( ij \) near the origin \( i \), when the congestion circle just encloses the destination \( j \), which can happen once during the congestion onset \( t^{c_{ij}}_{ij} \) and once during the congestion offset period \( t^{c_{ij}}_{ij} \). As such, the travel delay is correctly assessed during the periods of changing congestion.
3.3.2. Impact of sector overlappings on the link travel time

When the congestion circle radius is relatively large compared to the link length, the sectors can overlap. This complicates the calculations of the travel times, as the impact of a congestion circle on a link is determined by different effects. In Fig. 5b, e.g., an area emerges between origin and destination where the congestion area encloses the entire link: if the congestion center lies in the area labeled I+II+III, the maximum congestion circle radius is at least $r^m_k$. The entire link falls within the congestion circle with maximum radius. When the radius is larger than the link length, the link can even be entirely enclosed when the circle center lies outside the link (see Fig. 5c). Whatever the exact situation, as long as the link is not fully enclosed, the equations in Table 2 apply. The changes occur when the congestion area first encloses the entire link. At that time, full congestion already sets in, as further increases in congestion area are irrelevant for that particular link $(\text{cls}^k = d)$. This maximum congestion lasts for the duration the link lies completely in the congestion area.

Fig. 5. All possible congestion center locations with respect to the link $ij$. The maximum congestion circle radius is indicated as $r^m_k$. If the circle center falls outside of all indicated areas, then the congestion circle has no influence on this link travel time. If the circle center is located in sector $i$, then the congestion area remains between $i$ and $j$. In the other sectors, a part of the congestion area falls outside the link $ij$. The congestion area extends beyond $i$ in sectors II and III, and beyond $j$ in sectors IV and V. In sectors I, II and III, the perpendicular intersection point is located between $i$ and $j$. 
3.3.3. Impact of multiple congestion circles on the travel time on a single link

Multiple congestion circles coexisting on a single link requires careful bookkeeping regarding the magnitude and timing of the travel delays of the different congestion sources. When multiple congestion circles intersect with a link, first the separate travel delays are derived. Especially a correct determination of the pivot points in the travel time distribution is of key importance. In a second step, we propose to add the separate travel delays (see Fig. 13) as a first approximation of the true travel time delay occurring on that link. This means that travel delays are cumulative and do not increase computation time.

4. Implementation of the new network-consistent travel time layer

In Section 4.1 we discuss the required input data to generate a network-consistent time-dependent travel time layer. We also present a software implementation of our method. In Section 4.2 we show how a solution of a vehicle routing problem instance can be evaluated using a time-dependent travel time matrix generated by our method and discuss the effect of the network-consistent travel times on the quality of a solution.

4.1. Implementation

Recall from Section 3 that congestion circles are the key building blocks of the network-consistent travel time layer. As a result, the first decision to be made concerns the number of congestion circles that will be used in the analysis. Next, for each congestion circle, the location of the circle center and the maximum radius must be determined. This determines the links where congestion will occur. The time of congestion onset, full congestion, congestion offset and free flow phase of the congestion circles determine the time intervals when the enclosed link sections will be congested. If a sufficient amount of data is available in case of a real-life route scheduling problem, these time boundaries could be derived from historical databases. In an academic routing application, these time boundaries of the phases of congestion are randomly generated to comply with the following constraints:

$$
t_{con}^{c_k} \in \left[t_1^{cof, c_k}, t_1^{min, c_k}\right] \quad (11)
$$

$$
t_{off}^{c_k} \in \left[t_1^{min, c_k}, t_1^{hi, c_k}\right] \quad (12)
$$

$$
t_{cof}^{c_k} \in \left[t_1^{hi, c_k}, t_1^{con, c_k}\right] \quad (13)
$$

$$
t_{off}^{c_k} \in \left[t_1^{con, c_k}, t_1^{hi, c_k}\right] \quad (14)
$$

with \(t_1^{hi, c_k}\) the start (end) of a rush-hour period. An example template is provided in Table 3. All figures in the table are in minutes (distances and coordinates are also expressed in minutes of travel time).

Once all the information about the congestion circles is known, the time-dependent link travel time matrix can be calculated. This directed travel time matrix contains for each link a new \(\text{departure time, travel time}\) pair for each pivot point in the piecewise linear travel time profile. Generating data files that correspond to the typical situation with two rush-hour periods can be easily done by including a first set of congestion circles for which congestion onset is around the start of the morning rush our and congestion offset around the end of the morning rush hour, as well as a second set of circles (potentially in the same locations) for the evening rush hour.

The method presented in this paper to generate network-consistent time-dependent travel time matrices has been implemented in Java and is available online at \http://antor.ua.ac.be/network-consistent\, together with some instructions on how to use it. The tool processes two files: one containing information on node locations and the other on congestion circles, and generates a network-consistent travel time matrix based on this data.

### Table 1

Classification of the congestion circle scenarios according to their impact on the link travel time profile.

<table>
<thead>
<tr>
<th>Classification of the Congestion Circle Scenarios</th>
<th>Projection of the Congestion Circle Center</th>
<th>On Link</th>
<th>Outside Link</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum Congestion Area Encloses</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Origin to Destination</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Neither Link Boundary</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector I</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector II</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector III</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector IV</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sector V</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 2

Travel delay equations related to the congestion circle scenario.

<table>
<thead>
<tr>
<th>Sector</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>(t_{con}^{c_k} - TT_{off})</td>
</tr>
<tr>
<td>II</td>
<td>(t_{off}^{c_k} - TT_{off})</td>
</tr>
<tr>
<td>III</td>
<td>(t_{cof}^{c_k} - TT_{con})</td>
</tr>
<tr>
<td>IV</td>
<td>(d_{off}^{c_k} - TT_{con})</td>
</tr>
<tr>
<td>V</td>
<td>(d_{off}^{c_k} - TT_{con})</td>
</tr>
</tbody>
</table>

### Table 3

A template congestion circle input file.

<table>
<thead>
<tr>
<th>(t_1^{cof, c_k})</th>
<th>(t_1^{con, c_k})</th>
<th>(t_1^{off, c_k})</th>
<th>(t_1^{min, c_k})</th>
<th>(t_1^{hi, c_k})</th>
</tr>
</thead>
<tbody>
<tr>
<td>55</td>
<td>60</td>
<td>20</td>
<td>370</td>
<td>410</td>
</tr>
</tbody>
</table>

\(a\) From left to right, the table displays for every congestion circle \(c\) the x-coordinate of the circle center \((x_c)\), the y-coordinate of the circle center \((y_c)\), the maximum circle radius \((r_c)\), the time when the congestion circle first appears \((t_{con}^{c_k})\), the time when it first reaches its maximum size \((t_{off}^{c_k})\), the time when it first starts declining \((t_{cof}^{c_k})\) and the time when it first disappears \((t_{off}^{c_k})\).

\(b\) The \(X\) identifies that no travel delay occurs on a link in that scenario.

### Notes

- The symbol ** is used if the equation from sector I applies. The symbol \(\times\) is used if this variable is not applicable in that case.

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4.2. Results

In this section, we show the effects of the time-dependent travel times on the outcome of a vehicle routing algorithm. We first show the impact of a single congestion circle on the link travel times. Next, we demonstrate the impact on the (total) tour travel time when the best solution of a time independent (TID) setting is subjected to this new travel time layer. The results are based on Fig. 6. Impact of a single congestion circle on a road network. (a–f) Displays the travel times of the congested links of the Augerat et al. (1998) 32k5 VRP test instance, when subjected to a single congestion circle. All link sections inside the displayed congestion circle (c–f) are congested at the time indicated on top. The darker the shade of gray, the more substantial the travel delay. Note that the coordinates of the customer locations have been multiplied by 3.
on an Augerat dataset with 32 customers (Augerat et al., 1998). Note that the location coordinates are multiplied by three to make the link travel times more realistic at a free flow speed of 120 km/hour.

4.2.1. Impact of the new travel time layer on the link travel times

A single congestion circle (Table 3) is used to demonstrate the impact of adopting the newly proposed model to study time-dependent congestion effects. A graphical representation (Fig. 6) is provided which displays the links where congestion occurs. Every 50 minutes a new snapshot of the congested links is provided, starting at 4h50. Note that the starting time of travel delay due to a particular congestion circle depends on the direction of travel on a link, unless the circle center lies halfway between origin and destination. This must be considered when interpreting Fig. 6, as the single line connecting any link origin and destination can either be bidirectional or unidirectional in either direction.

Presented this way, the network-consistency of the congestion effects is easily verified. Even without knowing the exact location of the congestion circle, the intersection of the congested routes clearly indicates where the congestion (center) is located. The starting time and duration associated with the congestion circle determines when the congestion will be felt on the links. Links whose origin is far removed from the congestion area they pass through have to account for a travel delay even long before the congestion actually takes place (Fig. 6a). From then onwards, the travel delay on those links increases proportionally to the expansion of the congestion circle (indicated by a darker shade of gray in Fig. 6). The closer the link origin to the area of congestion, the later one has to account for a travel delay on that link. As a result, a higher number of congested links shows up in Fig. 6b and c. Evidently, congestion is felt immediately on all links leaving a location inside the congested area. Although the congestion area is at its maximum in Fig. 6d and e, fewer links have travel delays at the latter time. Indeed, for links with origins that are a long distance away from the area of congestion, the congestion will be dissolved completely by the time one arrives at the location of the congestion circle. This is particularly visible in Fig. 6f, where the long links still displayed are directed away from the congestion area. Indeed, all links with their origin situated close to or inside the congestion area will have travel delays. The short links displayed are faced with travel delays in both directions if both end points are close to or inside the congestion center.

4.2.2. Applying the new travel time layer to the vehicle routing problem

Network-consistent congestion effects play an important role when optimizing a fleet of vehicles. In a vehicle routing problem, typically the number of trucks (tours) needed to visit all customers is minimized and the customers are scheduled such that the total cost is minimal (Laporte, 1992). This cost can be expressed as a monetary, distance or time value. As it is common practice to minimise the distance, we will apply a travel time layer to the best known minimal distance solution of the 32k5 Augerat test instance (Augerat et al., 1998). Six congestion circles are generated to represent congestion in the network (see Table 5 in Appendix E for the congestion circle details). Fig. 7 provides a graphical representation of the location and timing of these congestion circles and of the minimal distance solution recalculated in terms of travel times taking into account these congestion circles.

Fig. 8a–d displays the projection of a vehicle routing solution on one of its location coordinates, where the horizontal axis represents the departure time and the vertical axis represents one of the location coordinates. It is of key importance to determine the impact on the (total) tour travel time when accounting for these travel delays. In the remainder of this section, we will demonstrate that the use of the congestion information protects against false expectations. Therefore, the minimal distance solution is compared against the time-dependent (TD) solution, when the same tours are subjected to the congestion effects (see Figs. 9–12).

The minimal distance solution is recalculated applying free flow conditions at all times. These time-independent (TID) (tour)
travel times are displayed in Fig. 8a and b. The congestion information is also displayed, as this information is available for the planner and can be used to deduce travel delay expectations. However, the arrival times at all customers might be misleading as no travel delays have been accounted for. In reality, travel delays will occur on the links crossing the congestion areas (thick lines in Figs. 7 and 8c and d). As a result, all arrival times at customers past the area of congestion are later when accounting for the congestion circles. E.g., from Fig. 8a and b, the vehicle is expected to reach customer 5 in a congested area during congestion offset. As customer 25 also lies in the same congested area, a major travel delay is expected when travelling from customer 25 to customer 5 and a minor delay is expected on the next link. However, earlier congestion on the links before customer 25 on that tour causes delays such that the vehicle reaches customer 5 in a free flow area, causing only a minor travel delay on the link (25, 5) and no travel delays on the next link. This finding is verified when comparing the travel times for all links in the tours in the time-dependent setting with the time-independent setting (summarized in Table 4). As in the time-independent setting the same congestion circle encloses the links (10, 25) and (25, 5) entirely, the same travel delay proportional to the free flow link travel time is expected (Fig. 8a and b; Eq. (1)). However, from Table 4 a travel increase of only 15.04% is observed for the link (25, 5), whereas a 67.19% travel increase was expected. The minor congestion expected on the link (5, 20) has dissolved completely by the time one starts on that link. Additionally, from Table 4 it is observed that all tours have travel delays due to the congestion circles, with a total delay of 240.17 minutes over all tours.

5. Conclusion and future research

In this paper, a novel method to generate time-dependent travel times was presented. Currently used travel time generation methods to generate vehicle routing problem instances do not necessarily adhere well to the desired characteristics of a travel time matrix.

In Section 2, a literature review revealed that only limited attention has been devoted to network-consistency, the paper by Kok et al. (2011) being a notable exception. We propose the use of circles with a fixed circle centre and variable radius to model traffic congestion. All links, i.e., straight line segments between node coordinates, that intersect with a congestion circle are included in the calculation of travel times.

\[ \text{Travel Time} = \text{Free Flow Travel Time} \times \left( 1 + \frac{\text{Congestion Delay}}{\text{Free Flow Travel Time}} \right) \]

This method ensures that travel times are consistent with the network topology, providing a more realistic representation of traffic conditions.

Fig. 8. Projection of a vehicle routing solution to visualise the impact of the congestion circles. In (a–d), the horizontal axis represents the departure time and the vertical axis represents one of the location coordinates. The congestion information has not been accounted for to calculate the travel times in (a and b), but has been accounted for to calculate the travel times in (c and d).
circle are affected with a travel delay proportional to the ratio of the length of the link section inside the congestion circle to the entire link length. All time-dependent travel time profiles are derived based on the congestion circle information, which makes this method conceptually elegant. The complete link travel time profile depends on the number of congestion circles affecting the link, the location of each circle center relative to the location of the link and the magnitude of each circle radius (see Section 3). As all link travel time profiles are based on the same set of congestion circles, the generated travel time matrix is guaranteed to be network-consistent. The amount of required input data to generate the travel time matrix is very limited (see Section 4), requiring only the location of each circle center and the time-dependent profile of each circle radius. As a result, different congestion scenarios are easily implemented for the same set of customers and depots. Therefore, the resulting travel time matrix can be viewed as an extra layer on top of the customer and depot location layer. A new \{(departure time, travel time)\} pair is stored whenever the slope of the travel time profile changes. As a result, no superfluous information is stored, while allowing an exact replication of the link travel time profile, which makes it attractive for routing optimizations. The applicability of the new method to vehicle routing applications was demonstrated in Section 4. We showed that the new travel time generation method accounts for the reality that links whose origin is far removed from the congestion area they pass through have to account for a travel delay even long before the congestion actually takes place or dissolves. It has been shown that not

![Travel time profile diagram](image)

**Fig. 9.** Travel time profile due to a single congestion circle: sector II (see Table 1). In (b), the lifecycle of the congestion circle is provided, with the departure time on the horizontal axis. The vertical axis represents the magnitude of the congestion circle radius. In (a), a graphical representation is provided to visualise the scenarios where the slope of the travel time profile on the link \(ij\) changes (provided in (c)), caused by the relative position of the congestion circle to the link \(ij\). Valid when the perpendicular intersection point between the congestion circle \(C_k\) and the link \(ij\) lies between origin and destination and the maximum congestion area encloses the origin. It is assumed that the traveler travels faster than the expansion of the congestion circle.
all congestion circles affect all links with the same intensity at any particular time. Congestion events close to the depot affect many routes, especially if they appear around the opening time of the depot.

The network-consistent time-dependent travel time layer described in this paper allows the design of algorithms and heuristics that take this special structure—that also exists in real life—into account and thus create more effective time-dependent routing methods. Additionally, dynamic variants of the time-dependent VRP can also be tested on the data generated by our method, e.g., to test an algorithm’s performance on non-recurring or unanticipated congestion.

An implementation of our method is available to generate the travel time profiles for all links in a road network based on a user-defined set of congestion circles. In this paper, we used a single value for the impact of a source of congestion on the travel time. An empirical study could reveal whether this value differs according to the road type. For example, an identical car incident could impact the travel time differently on a highway compared to a city street. In this work, we artificially generated the congestion circles. In line with the development of algorithms that use the special structure, we encourage research to derive the congestion circles from actual travel time data and relate the time-dependent circle profile to the underlying source of congestion (i.e., incident, bad weather, etc.).

Even though most research has focused on showing that congestion, and therefore time-dependent travel times, should be taken into account even in the absence of time windows at the customers, there is ample reason to assume that the presence of time windows will only make matters worse. A study of the effects

---

**Fig. 10.** Travel time profile due to a single congestion circle: sector III (see Table 1). In (b), the lifecycle of the congestion circle is provided, with the departure time on the horizontal axis. The vertical axis represents the magnitude of the congestion circle radius. In (a), a graphical representation is provided to visualise the scenarios where the slope of the travel time profile on the link \( ij \) changes (provided in (c)), caused by the relative position of the congestion circle to the link \( ij \). Valid when the perpendicular intersection point between the congestion circle \( C_i \) and the link \( ij \) does not lie between origin and destination and the maximum congestion area encloses the origin. It is assumed that the traveler travels faster than the expansion of the congestion circle.
of time windows on the importance of taking time-dependent travel times into account, is left for future research.

Appendix A. List of symbols

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x_a, y_a$</td>
<td>$x$-and $y$-coordinates of point $a$; a point is either a customer location, a depot location, or a mathematical construction such as the intersection of a congestion circle with a link</td>
</tr>
<tr>
<td>$c_k$</td>
<td>$k$th congestion circle</td>
</tr>
<tr>
<td>$v^p$</td>
<td>Perpendicular intersection point of a congestion circle with a link</td>
</tr>
<tr>
<td>$v^{m,a}$</td>
<td>Perpendicular intersection point of a congestion circle at maximum radius with a link $ab$, closest to point $a$</td>
</tr>
<tr>
<td>$d_{ab}$</td>
<td>Distance between points $a$ and $b$</td>
</tr>
<tr>
<td>$\gamma = c_k \overline{ij}$</td>
<td>Index for the effect of congestion circle $k$ on link $ij$</td>
</tr>
<tr>
<td>$\text{cls}_k(t)$</td>
<td>Length of the congested link section caused by congestion circle $k$ on link $ij$ at time $t$</td>
</tr>
<tr>
<td>$\text{cls}_k^m$</td>
<td>Maximum length of the congested link section caused by congestion circle $c_k$ on link $ij$</td>
</tr>
<tr>
<td>$TT_{ab}$</td>
<td>Uncongested travel time between points $a$ and $b$</td>
</tr>
<tr>
<td>$TT_{ab}(t)$</td>
<td>Time-dependent travel time at time $t$ between points $a$ and $b$</td>
</tr>
</tbody>
</table>

(continued on next page)
Maximum travel time between points a and b

Time of congestion onset of congestion circle \(c_k\) (radius is zero, starts increasing)

Time of full congestion of congestion circle \(c_k\) (radius is maximal)

Time of congestion offset of congestion circle \(c_k\) (radius starts decreasing)

Time of free-flow conditions of congestion circle \(c_k\) (radius is zero)

Start of a rush-hour period

End of a rush-hour period

Multiplication parameter for the travel delay on link \(ij\)

Maximum radius of congestion circle \(c_k\)

(Perpendicular) distance between the center of congestion circle \(c_k\) and the link \(ij\)

Appendix B. Calculation of perpendicular intersection point

Define \(x_i, x_j, y_i, y_j, y_c\) the \(x\) (\(y\)) coordinates of any customer \(i\), customer \(j\) and center of the \(k\)th congestion circle \(c_k\). \(r_{i,j}\) is defined as the radius of the \(k\)th congestion circle. \(v^p\) is the perpendicular intersection point between the link \(ij\) and perpendicular radius and the circle center, with coordinates \(x_{vp}\) and \(y_{vp}\). They can be calculated as follows:

\[
\begin{align*}
  y_{vp} &= r_c (x_{vp} - x_i) + y_i \\
  y_{vp} &= r_c (x_{vp} - x_j) + y_j
\end{align*}
\]

where \(r_1 = \frac{y_j - y_i}{x_j - x_i}\) and \(r_2 = \frac{x_j - x_i}{y_j - y_i}\). Solving for \(x_i\) in Eq. (15) yields:

\[
x_{vp} = \frac{y_{vp} - y_i}{r_1} + x_i
\]
substituting Eq. (17) in (16) yields:

\[ y_{ij} = r_{ij} \left( \frac{y_{ij} - y_i + x_i - x_j}{r_{1i}} \right) + y_c \]

\[ \left( 1 + \frac{r_{2i}}{r_{1i}} \right) y_{ij} = \frac{r_{2i} - y_i + r_{2i}(x_i - x_j) + y_i}{r_{1i}} \]

\[ y_{ij} = \frac{r_{1i}}{r_{1i} - r_{2i}} \left[ -r_{2i}y_i + r_{2i}(x_i - x_j) + y_i \right] \]

\[ y_{ij} = \frac{-r_{2i}y_i + x_i - x_j + r_{1i}y_i}{r_{1i}} \]

substituting Eq. (18) in (17) yields:

\[ x_{ij} = \frac{-r_{2i}y_i + x_i - x_j + r_{1i}y_i}{r_{1i} - r_{2i}} \]

\[ x_{ij} = \frac{x_i - x_j}{r_{1i} - r_{2i}} + \frac{y_i - y_j}{r_{1i}} + x_i \]

\[ x_{ij} = \frac{x_i - x_j}{r_{1i} - r_{2i}} + \frac{y_i - y_j}{r_{1i}} + x_i \]

\[ x_{ij} = \frac{x_i + x_{ij}r_{1i}^2}{r_{1i}^2 + 1} + \frac{y_i - y_j}{r_{1i}} + x_i \]

When \( r_{1i} = r_{2i} = 0 \) then one should rotate the axes over an angle \( \theta \neq 90^\circ \), with \( a \) an integer value. Every coordinate should be transformed using following equations.

\[ x' = x \cos \theta + y \sin \theta \] (20)

\[ y' = -x \sin \theta + y \cos \theta \] (21)

### Appendix C. Calculation of maximum radius intersection points

When the direct link \( ij \) crosses the congestion circle at maximum radius, the two intersection points can be calculated. Hereafter, we determine the intersection point closest to the destination \( j \), i.e., \( v^{n_j} \). We also know the length of the maximum congested link section \( \text{cls}^m \) (hereafter \( \text{cls}^m \)).

\[ y_{ij} = y_{ij} + r_{1i}(x_{ij} - x_i) \] (22)

\[ \text{cls}^m = 2 \sqrt{(x_{ij} - x_i)^2 + (y_{ij} - y_i)^2} \] (23)

where \( r_{1i} = \frac{y_{ij} - y_i}{x_{ij}} \). Substituting (22) in (23) yields:

\[ \text{cls}^m = 2 \sqrt{(x_{ij} - x_i)^2 + (r_{1i}(x_{ij} - x_i))^2} \] (24)

\[ x_{ij} = x_{ij} + \frac{\text{cls}^m}{2 \sqrt{(1 + r_{1i})}} \] (25)

### Appendix D. Special cases in the network-consistent travel time layer

See Figs. 9–13.

### Appendix E. Congestion information used in the Results section

See Table 5.
References


Fig. 13. Travel time profile due to multiple congestion circles. In (b), the lifecycle of the congestion circle is provided, with the departure time on the horizontal axis. The vertical axis represents the magnitude of the congestion circle radius. In (a), a graphical representation is provided to visualise the scenarios where the slope of the travel time profile on the link \( ij \) changes (provided in (c)), caused by the relative position of the congestion circle to the link \( ij \). Valid when the intersection between the link \( ij \) and the congestion circles \( c_k \) and \( c_l \) lie completely between origin and destination. It is assumed that the traveler travels faster than the expansion of the congestion circle.

Table 5

<table>
<thead>
<tr>
<th>( x_c )</th>
<th>( y_c )</th>
<th>( t_{con}^c )</th>
<th>( t_{con}^{f_{max}} )</th>
<th>( t_{con}^{f_{cof}} )</th>
<th>( t_{con}^{f_{ff}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>95</td>
<td>13</td>
<td>12</td>
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<td>30</td>
<td>8</td>
<td>590</td>
<td>630</td>
<td>650</td>
</tr>
</tbody>
</table>

* From left to right, the table displays for every congestion circle \( c \) the x-coordinate of the circle center \( (x_c) \), the y-coordinate of the circle center \( (y_c) \), the maximum circle radius \( (r_c^{\text{max}}) \), the time when the congestion circle first appears \( (t_{con}^{f_{max}}) \), the time when it first reaches its maximum size \( (t_{con}^{f_{cof}}) \), the time when it first starts declining \( (t_{con}^{f_{ff}}) \) and the time when it first disappears \( (t_{con}^{f_{ff}}) \).


