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Simulation with system dynamics and fuzzy reasoning of a tax policy to reduce CO_2 emissions in the residential sector

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Abstract

This methodological paper presents a planning and control methodology illustrated by a simplified case study on the carbon-tax design in the residential sector. The first objective is to show how to simulate with system dynamics the consumers' behaviour and the continuous tax-control mechanism depending on few important feedbacks, often ignored in static macroeconomic modelling. A second objective is to show how to aggregate external data driving this model and stemming from different sources with various credibility levels. This is realised by means of fuzzy-reasoning techniques incorporated into the system-dynamics model.

Keywords: System dynamics; Fuzzy reasoning; Environmental management; CO_2-emission reduction; Carbon tax

1. Introduction—Objective of the paper

Carbon-tax schemes may be developed to change the energy-consuming behaviour of residential consumers. They consist in applying a unit tax to each weight unit of fossil fuel to be burnt for producing electricity or heat in homes. Two basic aspects need to be addressed by the authorities when designing a carbon-tax scheme:

- The achievable and economically sound objective in percentage reduction of the use of fossil fuel.
- The value of the dynamic unit tax added to the price of fuel to approach the proposed reduction objective, without undesirable negative impacts on the economy.

There is a huge economic literature on optimal carbon-tax schemes to achieve some given abatement objectives and increase the macroeconomic welfare. Surveys are given in Nordhaus (1991), Boero et al. (1991), Cline (1992), Dean and Hoeller (1992). More recent papers are Azar and Schneider (2002), Weber et al. (2005) who developed a model in a system-analytical way, taking endogenous changes into account. As pointed out by Fiddaman (2002), many of the integrated climate-economy macromodels are equilibrium models which assume

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consumer and producer optimisation with full information and perfect foresight. In addition they assume instantaneous equilibration of factor inputs to optimal levels; they often neglect feedback mechanisms in the economy like important learning effect in technology development. The DICE model of Nordhaus (1993a,b) has been thoroughly discussed in Sterman (2002) from the point of view of material-flow conservation and the lacking feedback mechanisms. Relatively few integrated economy-climate models have been developed by means of system dynamics (SD). This simulation technique stresses feedback dynamics of stocks and flows and the associated time delays in achieving objectives and learning mechanisms. Some models are discussed in Fiddaman (2002) who compares DICE with a 'feedback-rich' climate-economy model called FREE, available on this author website. It results from this comparison that SD models focus on 'disequilibrium dynamics and feedback complexity, with behavioural decision rules and explicit stocks and flows of capital, labor and money', rather than on equilibrium and optimal factor allocations, in which there are "instantaneous tradeoffs between abatement costs and emissions" like macromodels do. An interesting paper using SD to evaluate the levying of a carbon tax in the goods transportation sector is Piattelli et al. (2002).

The objective of the model presented in this paper is not to develop a new economy-environment macromodel, but to take advantage of the properties of the SD technique to illustrate the development of a behavioural model taking into account data uncertainties.

We have chosen to model the behaviour of residential consumers submitted to the carbon tax, considering important feedback mechanisms, and exogenous technical and economic data essential for setting up the SD model. In this respect, an important econometric literature is available: Kaufmann (1994) (US), Smith et al. (1995) (eight OECD countries), Ekins (1996) (OECD countries), Vlachou et al. (1996) (electric industry in Greece); Fouquet (1998) (UK), Nomura and Akai (2004) (Japan), Brännlund and Nordström (2004) (regional effects in Sweden), to quote just a few. A difficulty in interpreting results from the econometric literature resides in the variety of analysis techniques, assumptions, geographical environments, social contexts and secondary benefits, as discussed in Howarth (in press). In addition, because only past data are used and get extrapolated into the future, the forecast results have to be taken with some care: this is especially true with still developing advanced pollution-abatement technologies. It is why there will be no lack of existing data sets for running the SD model, but rather a difficulty in reconciling their differences.

One basic variable is the marginal costs (MC) as a function of the achieved fuel reduction using advanced and partly untested technologies in the requested performance ranges. Sources are here technical as well as economic (Kunsch and Springael, 2005). Williams (1996) and Hope and Maul (1996) discuss some economic complex issues associated with MC; MC evaluations appear in the sustainability plan of the Walloon region (2003) in Belgium. Another question we will discuss in this paper to mimic the consumers' behaviour is the willingness-to-invest in such new technologies which may be influenced by feedbacks loops between increasing energy prices and the interest rates for loans. The willingness to pay for green electricity in Japan is discussed in Nomura and Akai (2004); demand for liberalised renewable electricity in UK is addressed in Fouquet (1998) and Howarth (2006) analyses social consumption effects of the carbon tax.

As all available econometric or modelling previsions on both aspects will generally not agree, there is an important source of uncertainty which must be taken into account when running the SD model. In the paper we discuss how the technique of fuzzy reasoning can be applied to cope with uncertainties.

The presented methodology is part of the more general framework coined 'Adaptive Control Methodology' (ACM) previously developed in Brans et al. (1998, 2002) and Kunsch et al. (2001). The ACM is a decision-aiding technique combining System Dynamics with multicriteria decision aid to assist policy-makers to make decisions in complex socio-economic systems. In this paper, we do not explicitly consider the multicriteria dimension of the decision problem, but we develop by priority the fuzzy-reasoning treatment of the time-dependent uncertain data used in SD modelling. The ACM also puts much stress on the need to periodically revise both data and model structure to take into account their real-world evolution.

The paper is constructed as follows.

In Section 2 we give the main principles useful for understanding tax schemes on fossil-fuel consumption to achieve CO_2-reductions. Section 3 describes the expected effect of the tax on the fuel consumption in the residential sector. Section 4 develops a simplified and deterministic system-dynamics (SD) model elaborated on these premises. Section 5 first discusses the nature of uncertainties on model parameters, before presenting how fuzzy-reasoning techniques can be used to handle these uncertainties in the SD model.

Conclusions are given in Section 6.

2. Principles of the fuel tax

There are different instruments developed to control pollution, in particular CO_2 emissions: the most pre-eminent ones are the marketable emission permits, and the carbon tax. A scheme of marketable emission permits functions on the basis of a 'cap and trade' principle: each year the regulating authority caps the yearly emission level by issuing the corresponding number of permits. Permits can be traded on a specialised market between the potential polluters. Kunsch and Springael (2005) have shown how to model marketable emission permits. Some results of this paper are used in the following to model the marginal-abatement cost curves, which are also needed in the tax scheme. We will not repeat these results here, and the reader is referred to this reference for more details.

In a tax scheme, the tax per unit of emitted CO_2, or CO_2-equivalent, is charged to the polluters. The principle is shown in Fig. 1. It is assumed that a curve (here represented by a straight line for simplicity) is perfectly known representing the marginal-abatement cost (MC) per unit of emission, in function of the total emission level of the specific pollutant (M).

In the classical handbook representation like in Hanley et al. (1997) and Perman et al. (2003), this curve would be decreasing to the maximum emission level per time unit (M_0), because of the law of diminishing return. M_0 corresponds to the situation where no abatement measure is taken. Considering the lower curve, a rational polluter would decrease the emission level from M_0 to M_T in order to achieve the economic optimum (see Perman et al. (2003) for a formal proof). The polluter has to pay the tax amount indicated by the rectangular surface. At equilibrium the total emission will be calculated as follows, given a tax level T^*, and inverting the function MC(M):

$$M_T = MC^{-1}(T^*). \tag{1}$$

Note that in the case of CO_2 emission in the residential sector, the practical way is to charge a so-called *energy tax* on the fossil-fuel emitting CO_2, rather than on the emissions themselves. The effects of the tax must thus be analysed on the basis of the consumer behaviour in reducing his consumption of fossil fuel. The presentation of the MC curve of Fig. 1 must be adapted to this situation:

- The horizontal axis 'r' now represents the reduction in fossil fuel consumption. The origin at $r = 0$ represents the initial conditions M_0 in Fig. 1 for which no reduction has yet taken place;
- For reasons of convenience, we assume a yearly reduction in quantity of fossil fuel, expressed in the units [wfuel/year] (where 'w' says for weight) equivalent to a unit reduction in CO_2 emissions, expressed in [tCO_2/year]. We thus have the following equivalence:

$$\text{1 wfuel/year less consumed is equivalent to} \\ \text{1 tCO}_2\text{/year less emitted.} \tag{2}$$

An important drawback of taxes is the difficult precise choice of the most efficient tax level for achieving some predefined objective M_T. Assume

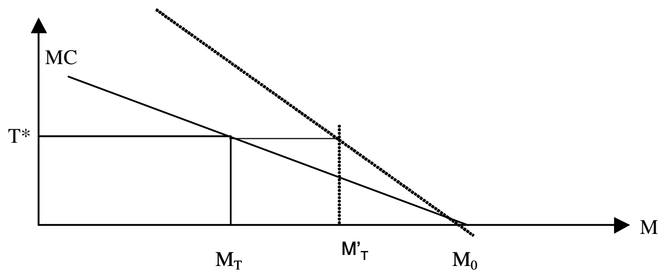

Fig. 1. Handbook representation of the marginal-abatement cost (MC) in function of the pollution level (M). T^* represents the tax level; for the lower line the optimal emission level for this value of the tax is given at M_T. The zero-abatement level is given at M_0. If the MC-curve changes to the upper line, the optimal emission level is increased to M'_T.

in Fig. 1 that the MC curve is moved upward due to uncertainties. The equilibrium emission level M_T will be translated to the right, giving a different policy result. It is why a sufficient degree of accuracy is needed in determining the MC curve. But as mentioned, many different evaluations regarding its precise shape may exist, and therefore there are important uncertainties.

3. Modelling a tax policy in the residential sector

In this section we assume that consumers are rational investors following the rules of cost-benefit analysis (CBA). Of course this assumption could be criticised, but it allows us to introduce our dynamic approach in a simple way. More refined assumptions are easily implemented in the simple investment model we now describe.

Call '*i*' the interest rate prevailing in the economy for borrowing money with the intention to invest into less polluting equipment in households. Elementary CBA teaches us that the cost of initial investment must be smaller or equal to the stream of discounted future benefits (Mishan (1988)). Thus:

$$\text{Cost of reducing one unit of fuel} = \text{Benefits of reducing one unit of fuel.} \tag{3}$$

(1) The cost of reducing one unit of (fossil) fuel is equal by definition to the marginal cost of reduction MC, expressed in currency units/year [CU/year], so that:

$$\text{Anticipated cost of fuel reduction } (t) = \mathrm{MC}(t). \tag{4}$$

Note here that the policy-makers scrutinising the abatement potential are not directly interested in the many individuals' MC curves. Rather, quite few MC curves of interest can be identified. They are common to similar techniques or technologies used by residential homeowners for reducing their fossil-fuel consumptions.

(2) The benefit of a unit reduction per year corresponds to an infinite stream of cost reductions attached to one reduced weight unit of fuel per year, the value of which is given by the well-known formula of perpetuity:

$$\text{Anticipated stream of benefits of fuel unit reduction per year } (t) = P_{\mathrm{w}}(t)/i(t), \tag{5}$$

where $P_{\mathrm{w}}(t)$ [CU/wfuel] is the unit price of fuel at time t including the tax; and $i(t)$ [%/year] is the yearly interest rate in the economy used for borrowing money.

By comparing Eqs. (4) and (5) we obtain the condition for the consumers to be willing to invest in fossil-fuel reduction or substitution measures in their homes:

$$\mathrm{MC}(t) \leqslant (P_{\mathrm{w/o}} + ut)(t)/i(t), \tag{6}$$

where $P_{\mathrm{w/o}}$ [CU/wfuel] is the unit price of fuel on the market at time t excluding the unit tax ut [CU/wfuel]. It is an exogenous variable to the model.

Eq. (6) is the basis of our model. MC curves when plotted in function of the reduction of the equivalent weight of fuel per year [wfuel/year] may have complex shapes, but sooner or later they are surging up to vertical. The economically achievable fossil-fuel reduction will be located in the bow to vertical. Fig. 2 shows the MC curve with these characteristics to be used later from Kunsch and Springael (2005).

Looking at this curve, it can be observed that a reasonable limit R for the yearly fuel reduction would be around $R = 50$ wfuel/year. This gives to policy-makers the information necessary to design a tax policy, assuming that they plan a time-horizon of T years to realise this reduction objective. Combining these two parameters thus provides a guideline $G(t)$ in [wfuel/year] for the dynamic reduction of fossil fuel $r(t)$ over T years. Assuming a linear evolution of the guideline with rate $g(t)$ we obtain a constant value:

$$g(t) = \frac{\mathrm{d}G(t)}{\mathrm{d}t} = \frac{R}{T} \text{ [wfuel/year}^2\text{]}. \tag{7}$$

This guideline will not be exactly respected because there are some delays in realising it, as will be shown later. It can be approached by adjusting the price at time t to its theoretical control value to match $G(t)$. This theoretical control price will be given by the following equality according to Eq. (6):

$$P_{\mathrm{th}} = i \cdot \mathrm{MC}[G(t)]. \tag{8}$$

The fossil-fuel price without tax $P_{\mathrm{w/o}}$ on the market will in general be fluctuating around a medium-term trend. The unit tax $ut(t)$ can be adjusted using proportional control, common in SD models, using the adjustment time T_{adj} as follows:

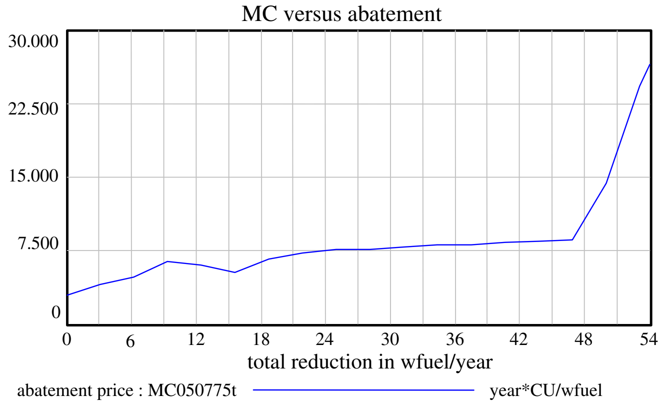

Fig. 2. MC curve from Kunsch and Springael (2005) representing the cost of reduction in [CU/(wfuel/year)] in function of the fossil-fuel reduction 'r' in [wfuel/year].

$$ut(t) = (P_{\mathrm{w}} - P_{\mathrm{w/o}})(t),$$
$$\frac{\mathrm{d}P_{\mathrm{w}}(t)}{\mathrm{d}t} = \frac{(P_{\mathrm{th}} - P_{\mathrm{w}})(t)}{T_{\mathrm{adj}}}. \tag{9}$$

Eq. (9) corresponds in SD to an exponential information delay of the first order, as shown in chapter 11 of Sterman (2000). Combining Eqs. (6) and (8) we can calculate the evolution of fossil-fuel usage. For that purpose, we first introduce the variable 'Willingness To Invest' (WTI) which expresses that the decision to invest passes the CBA test of Eq. (6): WTI is equal to:

$$= \begin{cases} 0 & \text{if } P_{\mathrm{w}} < i \cdot \mathrm{MC} \cdot (1-\varepsilon), \\ 1 & \text{if } P_{\mathrm{w}} \geqslant i \cdot \mathrm{MC} \cdot (1-\varepsilon), \end{cases} \tag{10}$$

where ε represents a tolerance for the CBA test, e.g. 0.5% is a reasonable value.

When $\mathrm{WTI} = 1$, investment is economically founded and it thus takes place according to our rationality hypothesis.

In our model we are not able to invert the MC curve (see Eq. (1)). To determine the reduction value $r(t)$ that will result from the use of a fuel price $P_{\mathrm{w}}(t)$, we use a calculation trick:

The evolution of $r(t)$ will be along the MC curve, so that, when $\mathrm{WTI} = 1$:

$$\frac{\mathrm{dMC}[r(t)]}{\mathrm{d}t} = \frac{\mathrm{dMC}}{\mathrm{d}r}\frac{\mathrm{d}r}{\mathrm{d}t} \to \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{dMC}/\mathrm{d}t}{\mathrm{dMC}/\mathrm{d}r}. \tag{11}$$

When $\mathrm{WTI} = 1$, the evolution of $\frac{\mathrm{dMC}}{\mathrm{d}t}$ is dictated by the price, with the evolution given by Eq. (9). A distinction must however be made between the two possible cases: either P_{w} is equal to (within the tolerance ε), or larger than $i \cdot \mathrm{MC}$:

While $\mathrm{WTI} = 1$,

EITHER $P_{\mathrm{w}} = i \cdot \mathrm{MC}$,

$$\frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\mathrm{dMC}/\mathrm{d}t}{\mathrm{dMC}/\mathrm{d}r} = \frac{\mathrm{d}P_{\mathrm{w}}/\mathrm{d}t}{i.\mathrm{dMC}/\mathrm{d}r}, = \frac{P_{\mathrm{th}} - P_{\mathrm{w}}}{T_{\mathrm{adj}} \cdot i \cdot \mathrm{dMC}/\mathrm{d}r}, \tag{12}$$

OR $P_{\mathrm{w}} > i \cdot \mathrm{MC}$,

$$\frac{\mathrm{d}r}{\mathrm{d}t} = g. \tag{13}$$

Thus in case P_{w} is larger than $i \cdot \mathrm{MC}$, the fuel reduction is dictated by the rate of change of the guideline in Eq. (7): the MC will move up to the value = (price/interest rate) at this rate of change. Two complications appear in this model:

(1) The increase of the total price with tax may be capped by the price-control regulator. Let us call $P_{\max}$ this maximum admissible value. This constraint is represented by adding to Eq. (9) a Verhulst-type factor (see Sterman (2000, p. 296)):

$$\frac{\mathrm{d}P_{\mathrm{w}}}{\mathrm{d}t} = \frac{(P_{\mathrm{th}} - P_{\mathrm{w}})}{T_{\mathrm{adj}}}\left(1 - \frac{P_{\mathrm{w}}}{P_{\max}}\right). \tag{14}$$

The value of $P_{\max}$ is subject to a decision of the authorities, which may require some form of political consensus. A simple way of determining this price is to read from the MC curve on Fig. 3 the MC value corresponding to the reduction limit R in Eq. (7), and to multiply it with the initial interest rate. In this particular case we would obtain $P_{\max} = 600$ CU/(wfuel/year), corresponding to the maximum economically feasible reduction $R = 50$ wfuel/ year, and $i_0 = 4\%$, assumed in our example.

(2) The increase of the fuel price to achieve the reduction guideline may have macroeconomic consequences on the willingness to invest, though the carbon tax is designed as to be revenue neutral as discussed in Mabey and Nixon (1997). The more expensive the fuel, the less economic growth is to be expected; the more expensive it will be for private investors to borrow money. A direct link is to be expected between the fuel price and the interest rate applied to loans. This creates an important positive feedback loop driving up the price still further. Without loss of generality we have represented in our model the impact of the fuel price on the interest rate as follows:

$$\Delta i_{\mathrm{P}} = f\left(I = \frac{P_{\mathrm{w}} - P_0}{P_{\max} - P_0}\right) > 0$$
$$0 \leqslant f(\cdot) \leqslant 1; \quad 0 \leqslant I \leqslant 1, \tag{15}$$

Δi_{P} is the change in the current interest rate from its initial value, due the change in fuel price with tax P_{w}. P_0 is the initial fuel price with tax I, calculated, as shown in the argument of the function $f(\cdot)$, called here the '*impact factor*'. The '*impact function*' $f(\cdot)$ is expected to be non-linear and increasing with the impact factor I.

4. System-dynamics modelling of the consumers' behaviour under the carbon tax

For simulation the SD code VENSIM DSS32© (1988–2003) is user-friendly, it has an easy-to-use graphical interface, and it works with vectors (subscripts), which is useful for dealing with several technologies and data sets. This code also has the capability of performing sensitivity analyses using Monte-Carlo analysis with given probability distributions of model parameters. The equations of this model are available from the authors on request.

Note that we do not use any real data for this simulation, the purpose of which is only explanatory.

First we start with the deterministic model:

The more ancient SD model of Kunsch et al. (2001) on the carbon tax in the residential sector

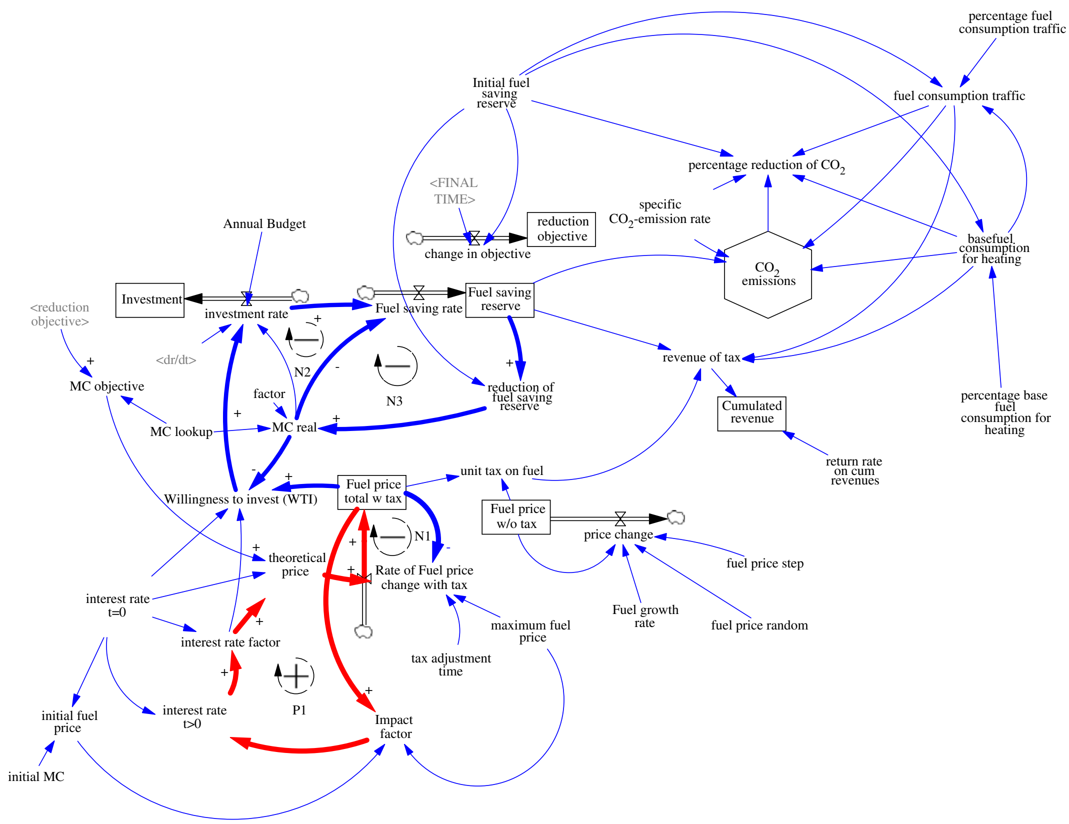

Fig. 3. The stock-flow diagram of the system-dynamics model of the carbon tax showing the main feedback loops.

was very simplified and deterministic. This first model has been expanded in the present paper to include the full tax model presented in Section 3. Of course, this more advanced model has still no claim of being complete. For example the contribution of traffic to CO_2 emissions appears only as an exogenous variable. The model mainly serves the purpose of being illustrative of our methodology, regarding the dynamic treatment of the problem and also the possibility of coping with uncertainties on the reliability of data coming from several sources, as we will discuss in Section 5.

The stock-flow diagram of the expanded SD model is reproduced in Fig. 3 (see Sterman (2000) for details on SD-modelling). It gives an overview of the tax model. Important feedback loops, which are represented in heavy lines, drive the tax-dynamics behaviour.

- The reduction objective, or rather the guideline, introduced in Eq. (7), is calculated in the upper part of the diagram. It functions as a feed forward in the model, triggering the adjustment in the fuel price by means of the tax. We have adopted in this simple example a maximum reduction $R = 50$ wfuel/year, as explained earlier in the text;
- Several stocks and three negative control loops are found in the central part of the diagram model: the price adjustment to the theoretical price described in Eqs. (8) and (9) (loop N1); the 'Willingness To Invest' according to the CBA test (10) (loop N2); finally the evolution of the fossil-fuel reduction along the MC curve according to Eqs. (11)–(13) (loop N3);
- The macroeconomic impact of the fuel-price increases according to Eqs. (14) and (15) is visible in the lower part of the diagram. It manifests itself by the existence of a positive destabilising loop (P1) on the interest rate (its initial value is chosen to be 4%/year);
- The calculation of the CO_2-emissions is in the upper right part of the diagram;
- The fuel price is expected to grow from its initial value at a rate of 1.25%/year (in real case studies available time series and scenario forecasts will be used).

In the SD model, three techniques are used in the residential sector to reduce the fossil-fuel consumption of private residents:

- RUE (rational use of energy), for example the use of long-lived and less consuming lighting bulbs, automatic light switches, etc.
- HED (high efficiency devices), for example double-panel windows, efficient boilers or refrigerators, etc.
- GE (Green electricity) use of renewable electricity (biomass, wind energy) for heating and lighting.

In the first deterministic calculation, we consider the MC-curve shown in Fig. 2 and the impact function in Eq. (15), which is represented in Fig. 4.

Fig. 5 shows some computation results with this SD-model and the described data. The upper left time graph shows the fossil-fuel reduction path lagging behind the linear guideline $G(t)$ defined in Eq. (7). This is due to the delays present in the system, e.g. in Eq. (9) corresponding to a first-order delay.

The upper right time graph shows how the fuel-price with included tax will have to increase, in order to follow as closely as possible the guideline according to Eq. (9). This price increase, expressed in [CU/wfuel], favours the investment willingness of homeowners, as shown in the bottom left figure representing the investment rate in [CU/year]. It is a rather erratic behaviour depending on the CBA test on the 'Willingness to Invest' (WTI) variable in Eq. (10). According to Eq. (15) another effect of the price increase is to bring the interest rate from its initial 4%/year value to higher values close to 6.5%/year, as shown in the bottom right graph. The effect of the positive feedback loop P1 evidenced in Fig. 3 brings in turn the price further up.

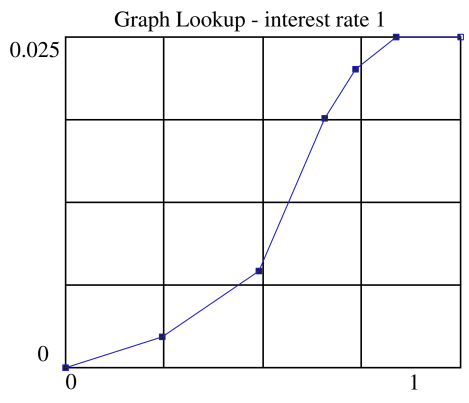

Fig. 4. Impact function $f(I)$ linking, as shown in Eq. (15), the impact factor I lying in the interval $[0, 1]$ to the increase in the interest rate Δi_P, with a maximum of 2.5%.

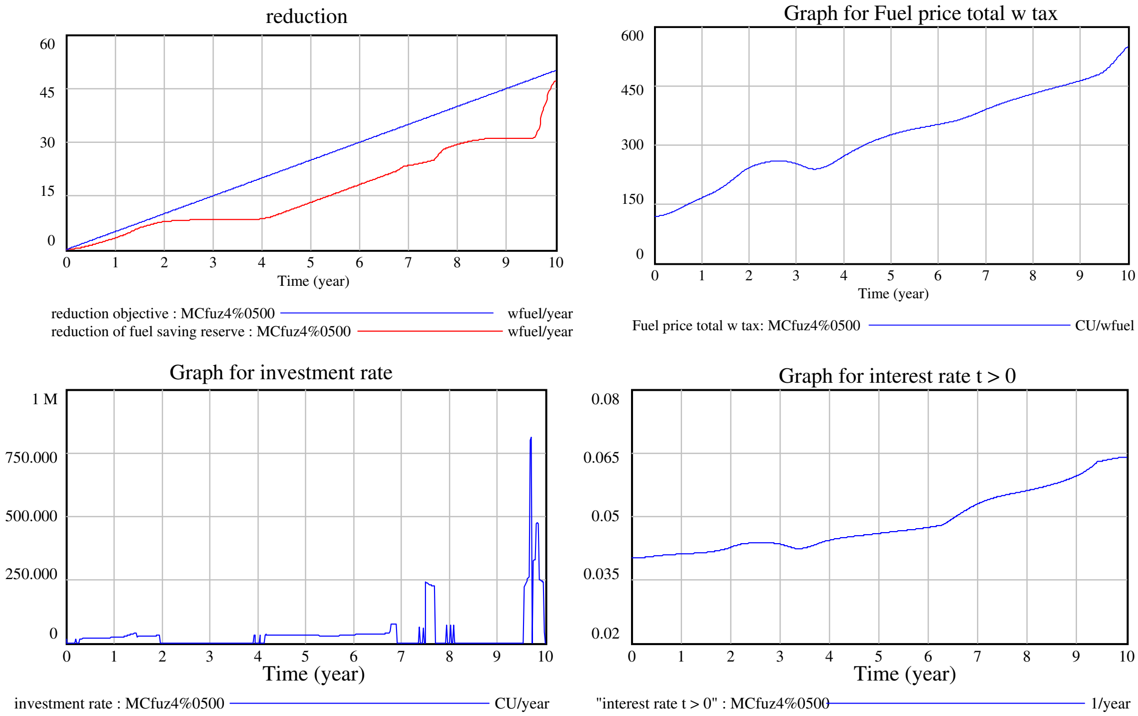

Fig. 5. Some results of the simulations with the SD-model of Fig. 3 using the MC curve of Fig. 2, and the impact function of Fig. 4. The straight line in the upper-left diagram represents the reduction objective.

5. Handling of uncertainties in the SD model

5.1. Generalities on parameter uncertainties

In the system, two types of parameter uncertainties are present in the SD model. They are not yet included in the deterministic model.

5.1.1. Static-parameter uncertainties

An example in the expanded SD model could be the total budgets available to achieve the reduction policy contained in the guidelines, described in Eq. (7). Simulation codes like VENSIM DSS32© (1988–2003) are able to generate results in the form of probability distributions of key variables in the model, evidencing percentiles (see also Fig. 9 introduced below). Assuming that some statistical distributions for those parameters can be defined, there would therefore be no difficulty in displaying such sensitivity results.

Also fuzzy reasoning can be used in cases when vague statements like: 'the budget is large' are available from the decision makers. This case will not be further considered here, but their inclusion in SD modelling is rather straightforward.

5.1.2. Dynamic-parameter uncertainties

Main examples of time-dependant parameters in our model are: the MC-curve in Fig. 2 and the impact function in Eq. (15). In general uncertainties on such parameter will manifest themselves by the existence of several results coming from different evaluations, data analyses, econometric forecasts, etc., all of which being more or less credible. A statistical treatment by scenario analysis of those different parameter assumptions is possible, but quite often the needed subjective probabilities are not easily available. Other means must be designed to aggregate the different results into one, to be used as an input to the model. The difficulty is to avoid loosing information in the process, while considering that some results or data are more reliable than others. We propose using fuzzy-reasoning based on rule systems to achieve this aggregation process.

Fuzzy reasoning was first developed in control theory. It can assist decisions on the basis of rather vague statements and logical rules (IF THEN) between variables. It is close to the natural language, this is why some people have labelled it 'computing with words'. It is very useful in many technical and economic applications in which stakeholders in a decision process express imprecise and relatively vague judgements. In the following we assume that readers are familiar with the basics in fuzzy reasoning. We only recall essential aspects. Useful references are Cox (1995), Passino and Yurkovich (1998), or Fuzzy Logic (2001).

Control theory takes advantage of an important property of fuzzy-rule systems: they provide universal approximations for any non-linear function whatever complicated it may look. This property is also used in Kunsch and Fortemps (2002) who use fuzzy reasoning with rule systems in cost calculations for nuclear-waste management. This property of fuzzy-rule systems is also used in the present paper. The intention is to aggregate data sets of different origins and of varying credibility to establish important dynamic parameters in the CO_2-model. The sources of data are the extrapolation of technical and economic data, usually made available by econometric models. Without loss of generality we can refer to any source of such prevision as 'data sets'. It will be only in rare cases that all data sets will fully agree on all their results.

For a choice of a fuzzy-rule system for aggregating data sets of various reliability degrees we use the conclusions of Kunsch and Fortemps (2004) in which two fuzzy-rule systems are analysed for the purpose of agent-based modelling: the well-known Mamdani–Sugeno inference, generally used in control theory, and the less familiar Kleene–Dienes inference. The conclusion is that the Mamdani–Sugeno inference is rather well adapted to 'smooth' aggregation and fitting of several functions (in this case preference functions, but other functions may be considered), while Kleene–Dienes gives rather 'brisk' results, in which jumps appear between functions to be aggregated. The latter system is thus better adapted to agent-based modelling in which clear-cut binary actions of the type GO/NO GO are meaningful. The former system better suits problems in which smoother changes between functions are needed, like in non-linear control theory. We think that it is also the case in our example, as it is needed to gently combine several data sets. We therefore adopt the Mamdani–Sugeno inference system in the following.

In a fuzzy-rule system, four steps come in sequence:

The first step is 'Fuzzification'. The basic ingredients of *fuzzification* are (1) 'membership functions' to represent a range of possible values of a vague or imprecisely known variable ('fuzzy variable' as opposed to 'crisp variable'), and (2) 'fuzzy rules'. The latter relate fuzzy variables, in the *antecedent* (or *input*) of the rule on its input side, to draw some conclusions on the final results, in the *consequent* (or *output*, or *conclusion*) of the rule.

The second step is 'Inference'. An *inference operator* defines the m.f.'s of consequents, given some value of the antecedent and applying the logical fuzzy rules. The 'min' operator, corresponding to a logical 'AND' is commonly used.

The third step in fuzzy reasoning is 'Aggregation'. In *aggregation*, the consequents of all partial rules are aggregated using an additional aggregation operator, usually the 'max' operator corresponding to a logical 'OR'. This operation will result in a composite membership function.

The fourth and final step is 'Defuzzification': it consists in deriving a unique crisp output from the fuzzy-rule system.

5.2. Fuzzy-rule system aggregating dynamic parameters

We now elaborate the fuzzy-rule system able to handle the uncertainties on dynamic parameters in the SD model. First we present the procedure in all generality; we then apply it for illustration to test the influence of uncertainties on two important dynamic parameters in the tax model.

Consider in the model some dynamic parameter 'DP' being the function of some other dynamic input variable 'DI'. Assume that an arbitrary number $n > 1$ of data sets of various origins are providing this function DP(DI). Their respective *credibility factors* (C_i, $i = 1, \ldots, n$) are given on a $[0, 1]$ scale. The provided functions in all sets need to be aggregated to obtain only one function reflecting all available information sources and their credibility factors.

In the 'Fuzzification' step, let us assume that the universe of discourse for representing the input DI to the rules is in the $[0, 1]$ interval (if it is not, rescale to this interval). We define L levels for the input variable DI, covering the $[0, 1]$ interval, each represented by a triangular membership function (m.f.) u_l, $l = 1, \ldots, L$. For reasons of convenience, we take $L = 5$:

$$(u_l;\ l = 1, \ldots, 5;\ \text{vanishing, small, medium, large, absolute}). \tag{16}$$

Each data set corresponds to a mapping of these DI levels to the levels of the dependent dynamic parameter DP. Each mapping generates a set of $L = 5$ membership functions, each representing an output O, i.e., here the dynamic parameter DP. This provides in all $n * L(=5)$ possible outputs, as follows:

$$O(i = 1,\ n;\ \ l = 1, \ldots, L; n \text{ data sets on the DP}(l) \\ \text{for } l = 1, \ldots, L \text{ DI} - \text{levels}) = O(i, l). \tag{17}$$

Outputs correspond to the set of $n * L$ partial rules:

$$\text{IF DI is DI}(l). \text{ AND. Data Set Credibility is } C_i \\ \text{THEN DP}(l) \text{ is } O(i, l). \tag{18}$$

Eqs. (16)–(18) complete ‘fuzzification’.

Let us now perform the second step: ‘Inference’ consists in calculating the m.f. of the conclusion of each partial rule using the Mamdani–Sugeno inference R_{MS}; this inference consists in obtaining the conjunction between the inputs and the output of the partial rule with the logical ‘AND’, represented here by the ‘min’ operator,.

Calling μ_{il} the m.f. of the conclusion of the partial rule (i, l) in Eq. (18), applicable to the lth level of the antecedent to the rule, we thus write:

$$\mu_{il} = R_{\text{MS}}(u_{il}, v_{il}) = \min(u_{il}, v_{il}), \tag{19}$$

where u_{il} represents the membership grade (m.g.) of the antecedent to the rule (i, l) in Eq. (18); v_{il} represents the membership function (m.f.) of the output $O(i, l)$ coming in the consequent of the rule in Eq. (18). The antecedent to the rule (i, l) in Eq. (18) is itself the conjunction of two inputs. The first one, we call it u_l, represents the m.g. of the lth DI-level (for example for DI$(l = 3)$, $u_3 = 0.4$), the second one represents the credibility C_i of the ith data set (for example $C_2 = 0.7$). Because this is a conjunction, the simple min-operator applied to these two values can be used: both values are by definition in the interval $[0, 1]$. In this case we obtain for u_{il} defined in Eq. (19):

$$u_{il} = \min(u_l, C_i). \tag{20}$$

(In this example $u_{23} = \min(0.4; 0.7) = 0.4$, for DI-level $l = 3$ (‘medium’) and data set $i = 2$).

Note from (20) that to a data set with a vanishing credibility will correspond a vanishing m.f. by application of the inference in Eq. (19). This data set will thus be ignored in the further data treatment. (If all data sets have vanishing credibility, no conclusion can be drawn at all from fuzzy reasoning).

For the m.f. v_{il}, defined in Eq. (19), the Sugeno inference, as a special case of the Mamdani–Sugeno inference, adopts a single value (i.e., a singleton with m.g. = 1). The Sugeno inference is able to represent arbitrary functions, the shape of which can be highly non-linear. The singleton receives the m.g. $v_{il} = 1$, so that Eq. (19) immediately simplifies to:

$$\mu_{il} = u_{il}. \tag{21}$$

In the Sugeno inference the consequent of each rule thus receives the same m.g. as the antecedent of the rule.

The next step is ‘Aggregation’. The conclusions of all partial rules (i, l) relative to all data sets and all DP levels are combined in order to obtain a global m.f. This is done by using the logical ‘OR’, represented here by the ‘max’ operator applied to the consequents of all individual rules. In the Sugeno inference this is particularly simple, because the conclusion of each rule is a singleton: the m.g. has just been calculated in the previous inference step.

The global aggregated m.f. has thus $n * L$ components given by

$$\text{m.g.}(I) = \{u_{il}; i = 1, \ldots, n; l = 1, \ldots, L\}. \tag{22}$$

The final step in the fuzzy-reasoning schemes is ‘Defuzzification’ of the global m.f.’s of the DP. It consists in calculating a unique ‘crisp’ value from the global m.f. for the DP. In this particular case the center of gravity (COG) is the most adapted approach to obtain this value. This is expressed as follows for each set of inputs (impact factor and credibility factors):

$$\text{COG}[\text{DP}(l), l = 1, \ldots, L; C_i, i = 1, \ldots, n] \\ = \frac{\sum_{i=1,\ldots,n;l=1,\ldots,L} O(i, l) u_{il}}{\sum_{i=1,\ldots,n;l=1,\ldots,L} u_{il}}. \tag{23}$$

This is the final aggregation formula for the dynamic parameter function DP(DI). Note that it is also valid when $n = 1$, because of the property of universal approximation of fuzzy-system rules. It is then shown to extrapolate between the ‘anchor values’ for all L values of the dynamic input variable DI.

5.3. Use of the fuzzy-rule system to account for uncertainties in the SD model

We now illustrate for $n = 2$, how to include the uncertainties on two main dynamic parameters in the SD model of Fig. 3.

5.3.1. Aggregation of marginal-cost curves

Kunsch and Springael (2005) provide as an illustration the aggregation of two MC-curves with respective credibility $C_1 = 0.5$ and $C_2 = 0.7$. Each dynamic curve is assumed to take learning effects into account. A complication arises because of the existence of the three technologies RUE, HED, GE, and the need to collapse three MC curves into one. We use this result in the present SD model with uncertainties without further comments.

5.3.2. Aggregation of the impact function

The fuzzy-rule procedure is now applied in details to the dynamic parameter $\mathrm{DP} = \Delta i_{\mathrm{P}}(I)$, the change in interest rate in function of the impact factor I, as given by Eq. (15).

Again $n = 2$ and $C_1 = 0.5$ and $C_2 = 0.7$. Thus in addition to the impact function of Fig. 4 (1st data set), a (very different) second function (2nd data set) is provided in Fig. 6. The fuzzy-reasoning process can be followed step by step in Fig. 7.

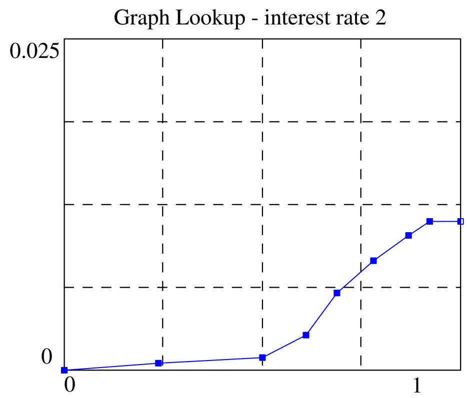

Fig. 6. $n = 2$; $C_1 = 0.5$ and $C_2 = 0.7$. A second interest-rate impact function $\Delta i_{\mathrm{P}}(I)$ of the impact factor I in Eq. (15) is provided, in addition to the function given in Fig. 4.

Because $n = 2$ and $L = 5$, there are 10 rules, each represented in a separate window. The membership grade of the combined inputs is calculated by means of the conjunction operator 'min' on the left. The membership functions of the conclusions on the right reduce to singletons, which receive the same membership grade as the combined inputs of the applicable rule. These singletons are obtained for each data set by considering the five 'anchor values' $(I(l); l = 1, \ldots, 5)$ of the impact functions of Fig. 4 or 6.

The lowest frame on the right of Fig. 7 represents the set of values constituting the global m.f. The latter aggregates all partial rules. The calculation of

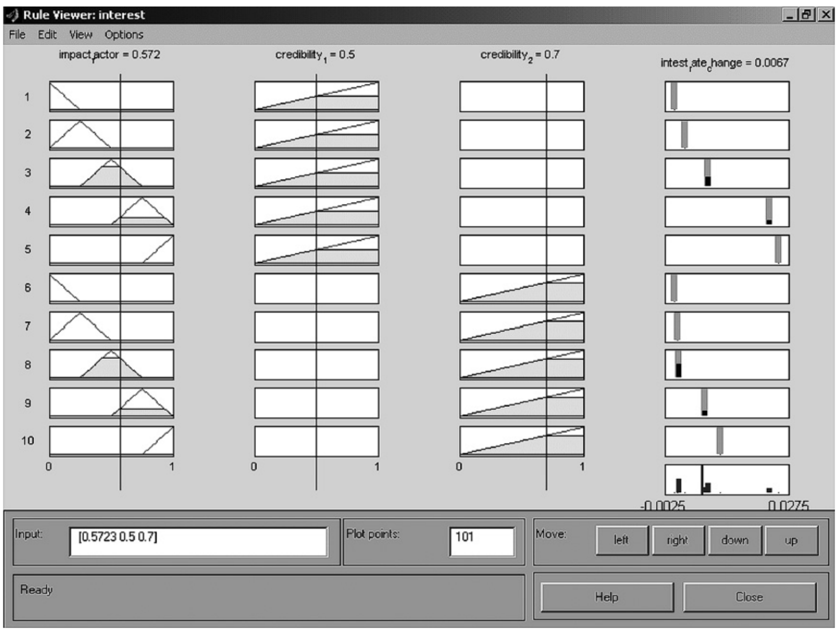

Fig. 7. The fuzzy-reasoning steps on the impact function defined in Eq. (15) for $n = 2$ ($C_1 = 0.5$; $C_2 = 0.7$) are from left to right: 'fuzzification' of three inputs (I, C_1, C_2); 'Inference' of the partial Sugeno-rules, 'Aggregation' of the partial conclusions of rules to a global m.f. (right-bottom window), and 'defuzzification' of the global m.f. to a unique output through the center-of-gravity methodology (=0.67%) (Arbitrary scales and units). (Calculations are made with Fuzzy Toolbox of MATLAB® (2001).)

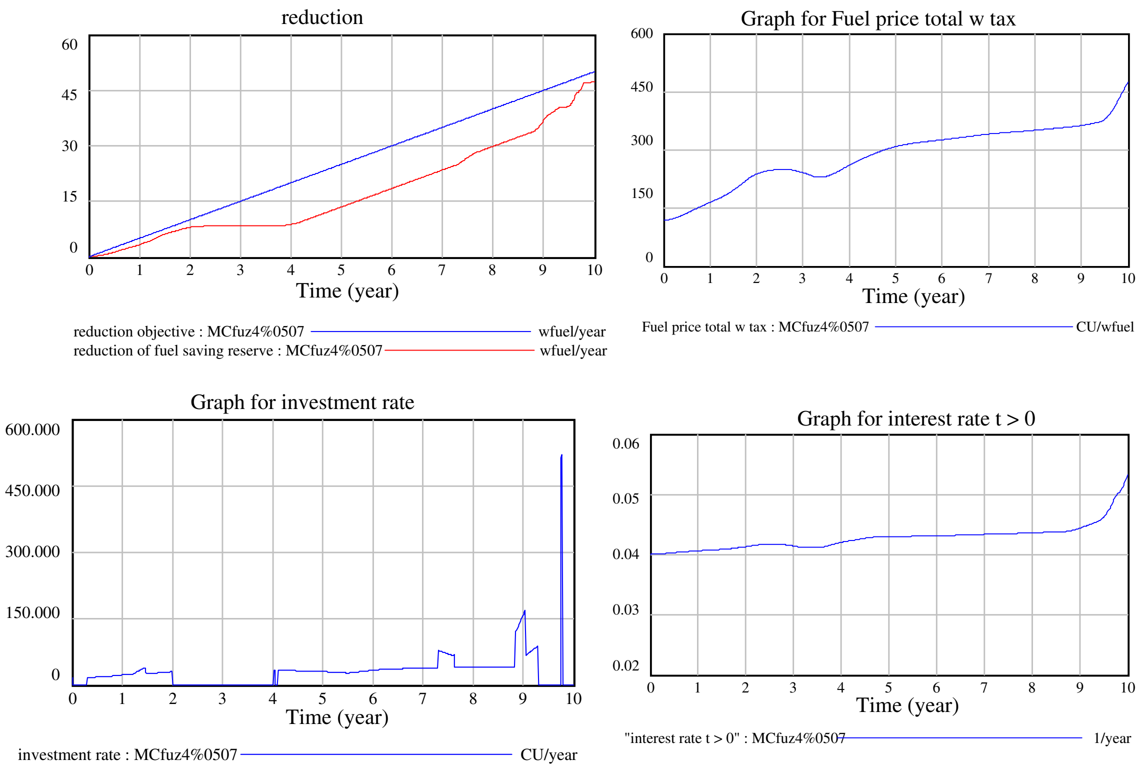

Fig. 8. The results of the simulation for $n = 2$ ($C_1 = 0.5$; $C_2 = 0.7$) for the impact functions of Figs. 4 and 6 and the aggregated MC-curves from Kunsch and Springael (2005). (This figure is to be compared with Fig. 5 which is established for $n = 1$ with $C_1 = 1$.) The straight line in the upper-left diagram represents the reduction objective.

the COG is shown in the lowermost window on the left of Fig. 7, giving a current value $\Delta i_P = 0.67\%$ for the increase of the interest rate.

At each time step, a fuzzy-reasoning process takes place in the way shown in Fig. 7, which is static. For our purpose it is easily made dynamic by combining it with the SD model presented in Fig. 3: in each time value, the computed values of the variables from the previous time step are used for forward computing in the following time step. For a more general model, the assumptions on the number of Data Sets and technologies are introduced as adaptable parameters in the models. Also the assumption on constant credibility factors can be easily relaxed as discussed later in Section 5.4.

Using the subscript technique and lookup tables available in VENSIM DSS32© (1988–2003), a fuzzy-reasoning module has been added to the SD model in the case with $n = 2$ that we have presented here. Of course the same technique can be used for any value of $n > 1$.

Fig. 8 shows the final results with the two ($n = 2$) data sets ($C_1 = 0.5$; $C_2 = 0.7$).

5.4. Sensitivity analysis on results

The comparison of Figs. 5 and 8 reveals important result differences between the deterministic ($n = 1$) carbon-tax model and the extended model including the fuzzy-reasoning procedure considering several data sets to be reconciled (here we used only $n = 2$ for illustration).

Though the eventual reduction path comes out to be nearly the same, the price policy, and thus the tax evolution prove to be quite different. These results stress the overwhelming importance, when designing an efficient tax scheme, of having a sufficient knowledge of exogenous parameters used in the SD model, and to assess their sensitivities with respect to multiple sometimes-contradictory external evaluations.

Of course, in general credibility factors allocated to the multiple data sets are not precisely known: in

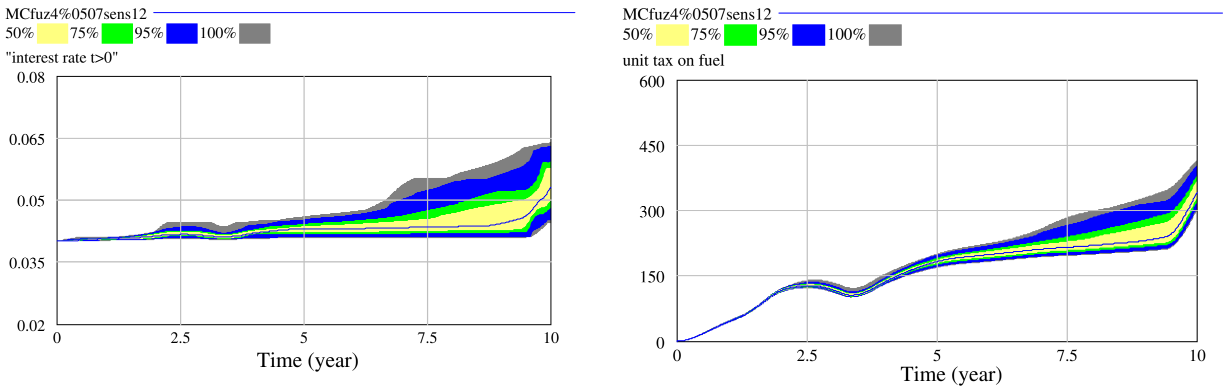

Fig. 9. Sensitivity analyses on the credibility factors. Several percentiles of the represented stochastic variables are displayed (on the left: interest rate; on the right: unit tax on fuel), assuming that both credibility factors are uniformly distributed in the range [0, 1]. In each figure the central trace corresponds to the base case ($C_1 = 0.5$; $C_2 = 0.7$).

the given example, the chosen values for the assumed base case would be highly debatable in a real case study.

A multivariate sensitivity analysis has to be performed, assuming that there is some agreement among decision-makers on defining intervals in [0, 1] for the C_i's values. Calculating individual trajectories (traces) with possible values, using the SD code, is straightforward.

As an example Fig. 9 shows several percentiles (50%; 75%; 95%; 100%) of the distribution of the interest rate (left) and of the unit tax on fuel (right) when simulating 200 traces; it has been assumed in this sensitivity analysis that nothing at all is known about the credibility-factor values, so that the latter are considered as being random variables uniformly distributed in the interval [0, 1]. The central trace in each graph corresponds to the base case, i.e., $C_1 = 0.5$; $C_2 = 0.7$. Traces can be individually plotted, so that a decision on the carbon tax can be made within a defined confidence interval.

6. Conclusions

In order to design a coherent and efficient strategic plan for a CO_2 tax scheme over a medium-term horizon, e.g. five years, decision-aiding instruments must be developed. It is the purpose of our approach to assist and formalise the preparation of a strategic plan, considering the dynamic and structural aspects, not addressed in static models. These aspects have been handled with system dynamics, which puts forward important feedback loops describing the effects of the tax on the CO_2-reduction behaviour of consumers and vice-versa.

Another difficulty beyond the dynamic aspects is that a deterministic CO_2-control model is not sufficient for strategic planning. In the model one cannot ignore that there are uncertainties on time-dependent key parameters or exogenous variables in the SD model.

A first proposal is to develop scenario and sensitivity analyses to address the uncertainty issues. Classical scenario analysis is not entirely satisfactory, however. It provides ranges of values, elaborating on optimistic, average, or pessimistic forecasts related to the evolution of the output variables in the model. But collapsing many scenarios into one requires an additional knowledge of a priori occurrence probabilities, which is almost never available.

In the present paper, we propose to use instead fuzzy reasoning based on rules with the idea to combine several available, sometimes partly contradictory, previsions on these dynamic data sets. In technically complex frameworks, like CO_2-emission control in the residential sector, past data, especially in the field of technology development or macroeconomic behaviours, cannot easily be extrapolated into the future, or at least there are large uncertainties. The concept of 'probability', central to the Bayesian approach of subjective probabilities is difficult to transpose to those cases. Instead imprecise semantic statements like 'small', 'large', etc., typical for fuzzy reasoning, often better capture the uncertainties in the problem. The central value of fuzzy-rule systems will be to keep all available information and to merge available data sets according to their respective credibility factors. There will be in those conditions no need for a-priori subjective probabilities, like in the Bayesian approach. It will only be assumed that a scoring on a $[0,1]$ scale is available for representing the credibility of the available data sets used for the uncertain parameter determination. This scoring may be derived for example from the measured accuracy of past technical or economic forecasts on those matters. It is beyond the scope of this paper to discuss how the scoring within the fuzzy rules can be established. The readers are referred to the existing literature. A valuable reference is Meyer and Booker (2001) for scoring expert results. Note that it can be decided to use cut-off rules to eliminate the less credible results, for example deleting all data sets with a credibility factor less than 0.25, etc. The same approach could of course be used in principle by directly attaching credibility scores to scenarios for available historical data, but this seems to be a more perilous attempt, because the Past is only a poor predictor of the Future. One more added value of fuzzy aggregation formula (23) comes from that it does not only interpolate between anchor points. It provides in addition an easy approach for aggregating all available data sets to one global result. Regarding the use of credibility scores, this approach is thus far less arbitrary than a simple additive weighing technique. We also have shown in Fig. 9 that uncertainties on the credibility factors can be handled with Monte-Carlo analysis.

To account for uncertainties on any static parameters in the model, the same type of approach can be used. To cope with uncertain futures, fuzzy reasoning alone will not suffice in the long term, however. The mentioned ACM procedure from Brans et al. (1998, 2002) recommends revisiting periodically, for example every five years, not only the data, but also the model and its structure. Comparing the strategic previsions to the observed evolution, and making the necessary adjustments is the preferred approach.

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