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Towards Statistical Multicriteria Decision Modelling: a First Approach

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ABSTRACT

Many real life situations result from decisions taken by a very large number of decision makers. Among them, we may cite road traffic congestion, crowding during shopping, equity market behaviour, distribution of holiday destinations, etc. Furthermore, these decisions often depend on the optimisation of several conflicting criteria. In this paper, we introduce a new multicriteria tool based on Markov chains to model and manage these macroscopic phenomena. Finally, the road traffic congestion problem will be considered to illustrate the applicability of our approach. Copyright © 2003 John Wiley & Sons, Ltd.

KEY WORDS: multicriteria decision aid; statistical modelling; Markov chains; road traffic congestion

1. INTRODUCTION

One of the most recent developments in the field of decision aid as a subdomain of operational research is that of group decision making. The modelling and management of decisions taken by several decision makers has been, for the last decade, the center of interest of a still growing field of research. In combination with tools developed within the multicriteria decision aid community, new approaches were born and have led to both interesting theoretical and practical results (see e.g. Macharis et al., 1998).

In this paper, the authors consider a problem beyond usual multicriteria group decision making in the sense that the number of decision makers involved (e.g. 100 000 decision makers) is such that it is impossible to model or even to observe each individual one. Such situations are frequent in many real life situations. Let us cite for instance road traffic congestion, the occurrence of queues during shopping, equity market behaviour, distribution of holiday, etc. In these extreme conditions, classical group decision tools are technically not applicable.

Therefore, the authors have developed the idea of statistical multicriteria decision modelling (SMDM), in which the objective is not to find a unique or a set of good solutions for a given problem. It is rather to model decisions taken by a very large number of decision makers, i.e. to obtain for each potential action a theoretical frequency of related decisions for the considered group. This tool will be of great help to model and manage situations such as those mentioned above. Furthermore, it is worth commenting that our approach is neither prescriptive, nor supportive as it is the case for group decision support systems, but purely descriptive.

In the next section we present a possible conceptual framework to tackle the problem. Intuitively, each individual will make a decision on the basis of his own preferences. From a global point of view, all those elementary decisions give rise to so called decision frequencies. This phase will be modelled by means of Markov chains. Our approach is based on the assumption that the individual preferences of a large group of decision makers can be modelled through a unique, though sufficiently general, preference matrix.

The Markov chain technique is extensively used in marketing management. Most analyses concentrate on predicting future consumer choices in purchasing products from different competitors. The purpose is to forecast the magnitude and speed of change in future market shares given the

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present market shares (Kotler, 1984). Other applications deal with interactive marketing to establish the budget of marketing expenditures, see for example Pfeifer and Carraway (2000). In all these applications the stochastic transition matrix is directly available from market surveys. In the present paper we establish an approach to derive the relevant transition matrix by using multicriteria modelling techniques. To our knowledge, this has never be attempted before. We feel that it could also be a benefit to marketing management. A review of the use of the Markov chain techniques, including in marketing management, is given in White (1993).

The illustrative example treated in the third section is that of road traffic congestion generated by the decisions of car commuters. The decision involved here is the determination of the departure time, which is based on a (unconsciously made by road users) multicriteria analysis involving criteria such as 'time spent on road', 'wake up too early', 'penalty for late arrival on job', etc. After a short introduction, the structure of the simulation model is described. This section terminates with some simulation results including different policy measures and the differentiation of two large social groups (labourers and office workers) on which policies have different impacts.

The objective of the paper is to derive an original methodology, and to show its practicability with a simplified notional example. Though the latter is not coming from real practice, it is close enough to the concerns of contemporaneous decision-makers. However, it could be expanded later to make it more realistic.

We end this paper with some conclusions, remarks and open questions.

2. THE MODEL

Let us consider a multicriteria decision problem characterized by a set of p potential actions, $A = \{a_1, a_2, ..., a_p\}$, and a set of q criteria, $G = \{g_1, g_2, ..., g_q\}$. Let $D = \{d_1, d_2, ..., d_r\}$ be the set of decision makers facing the choice problem based on the previous framework. We assume that the cardinality of D is such that it is too difficult, or even impossible to represent or observe every individual decision.

Due to the multicriteria nature of the problem, each decision maker chooses the action that fits best his/her own preferences. Being heterogeneous, two decision makers may choose two different actions. Hence, from a macroscopic point of view, we will not observe a unique decision but rather decision frequencies $\{fr_1, fr_2, ..., fr_p\}$, where fr_i is the observed frequency related to the choice of action a_i. The aim of the proposed method is to model the decision process that leads to these frequencies. This will for instance allow us to study impacts, on the macroscopic behaviour of decision makers, when adding new potential actions or modifying existing ones.

The main underlying assumption of the proposed method is that the decisions taken by the group of decision makers D can be modelled through a discrete stochastic process on the set A, denoted $\{X_n,\ n \geqslant 0\}$. The intuition behind this assumption is that for each decision maker choosing an action is a dynamic process. Suppose that a_k is the current best action for decision maker d_i. Before definitively choosing a_k as the best one, d_i will compare it to all other actions. Based on preferences between a_k and any other action, d_i will decide to reconsider his choice or not. Each time a new action is considered the comparison process restarts. Our model assumes that:

$$P(X_{n+1} = a_l | X_n = a_k) = p_{kl} = F(\pi_{lk}, \pi_{kl}) \tag{1}$$

with π_{kl} the valued preference of a_k on a_l and F a normalized distribution function, to be defined. In other words, we impose that the probability to choose a new action a_l instead of the current one a_k only depends on the preferences between these two potential actions. If m decision makers are considering that a_k is the current best action, only $m \cdot F(\pi_{lk}, \pi_{kl})$ will reconsider their status in favor of a_l. The selection process can therefore be represented by a Markov chain. To simplify the model, we have supposed that each decision maker is using the same model, i.e. the same stochastic process and the same preference matrix.

Assuming that this Markov chain is well adapted to describe the decision process of all the decision makers $d_i \in D$, its stationary distribution $(p_1, ..., p_p)$ will provide the frequencies $\{fr_1, ..., fr_p\}$.

As a matter of fact, the construction of the preference matrix comparing every pair of actions $\Pi = \{\pi_{ij}\}$ $(0 \leqslant \pi_{ij} + \pi_{ji} \leqslant 1)$ is a critical step of our approach. This matrix can be obtained, for instance, by applying the PROMETHEE (Brans and Mareschal, 1994) formalism. Π being unique, it is supposed to be sufficiently general to represent all the decision makers preferences. Therefore, its computation must take into account the different opinions present in the group. Weights and preference thresholds, if used, have to be carefully chosen. This part of the model is of course quite sensitive and must be adjusted and validated through the obtained results.

Until now, we have no theoretical constraint to define the function F, but as a first sensible attempt we propose to define it in the following manner:

$$p_{kl} = F(\pi_{lk}, \pi_{kl}) = \begin{cases} \dfrac{\pi_{lk}}{p-1} & \textit{if } k \neq l \\ 1 - \sum_{k \neq j} p_{kj} & \textit{otherwise} \end{cases} \tag{2}$$

This transformation implies that the probability to choose an action l instead of the current action k is directly proportional to the preference of action l on action k. In addition, this probability is weighted by $1/(p-1)$ which can be interpreted as the uniformly distributed probability to choose one action among the $(p-1)$ remaining alternatives.

It is obvious to notice that many other transformations F can be considered which is a direction for ongoing and future research.

A desirable property for F is that by applying it to $\{\pi_{ij}\}$ the obtained Markov chain is irreducible and ergodic. This constitutes a sufficient condition to obtain an unique stationary distribution (Ross, 1997).

Let us remark that Glineur (1998) has already considered the application of Markov chains to multicriteria problems. However, his work is dedicated to the choice of a single alternative among several. In multicriteria terms, such approach is categorized under the so-called ‘choice problematic’. Unlike the other approaches within the field of MCDA, our intention is rather to perform a descriptive analysis of a decision problem involving a large number of decision makers, i.e. to describe the decision frequencies over the set of *all* alternatives and to study how they can be affected by different factors.

3. AN EXAMPLE: THE ROAD TRAFFIC CONGESTION PROBLEM

Let us note that the example, presented here below, has an illustrative purpose. Our goal is to demonstrate the applicability of the framework rather than giving a complete detailed analysis of the road traffic congestion problem.

3.1. Introduction

Nowadays one of the major challenges for policy makers in the Western countries consists in handling the traffic congestion problem around the cities. Several models have been developed to study potential policy measures in order to influence the density of car commuters on the roads.

An early approach in which the car commuters’ behaviour was explicitly taken into account was presented by Small (1992). In this study the car commuter is supposed to make a trade-off between different cost-functions (criteria) representing the time to be too early or too late on his/her job and the time spent on road when driving from home to office. Each commuter is then supposed to determine his personal strategy. Knowing the private marginal cost as a function of time (Mirabel, 1997), the model is reduced to an optimization problem in which the distribution of arrival times has to be optimised, taking into account several restrictions. By doing so the model optimises the collective utility and finally gives rise to a distribution of arrival times (see Figure 1).

In order to internalize the externalities due to traffic congestion a toll was introduced during the peak hours. A second policy option might be to

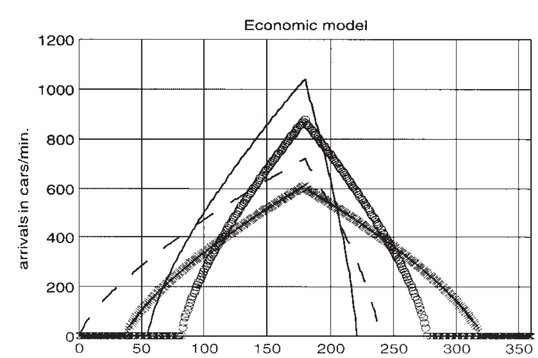

Figure 1. Results of the simulation for the distribution of arrival times using optimal control theory. The highest arrival distribution shows the ‘no policy case’ (——). The three other distributions progressively flatten as the policies are applied in the following order: ‘toll alone’ (— — —), ‘flexible time alone’ (oooo), ‘toll and flexible time together’(xxxx) (from Kunsch et al., 2001).

introduce flexible working hours. Applying the latter, people may arrive (and leave) earlier or later at their job, inducing a widening of the distribution of departure times and a decrease in amplitude and consequently a decrease of the density of people on the road.

Using optimal control theory, the following results were obtained (see Figure 1) (Small, op. cit.). As shown in this figure the introduction of a toll will spread out considerably the peak-hour time if combined with flexible working hours.

Another more extended and dynamic approach was presented by two of the authors: Kunsch et al. (2001) and Springael et al. (2002), taking into account several criticisms on the initial economic model such as

1. the use of costs in order to determine the strategy while most criteria are defined on a qualitative scale;
2. the absence of the dynamic aspects in the determination of the several strategies (learning effects, etc.).

We invite the interested reader to consult the reference Springael et al. (op. cit.) for a detailed description of the extended model. Though we would like to use this example in order to illustrate the theoretical formalism of the previous section and the applicability of statistical multicriteria decision modelling (SMDM).

3.2. Assumptions of the simulation model

In this example we start from the same assumptions as in previous models. We consider a single road on which 10^5 car commuters must drive from their home (a single place) to the central business district (another single place), between 6 and 12 a.m. Hence, these commuters have to make a decision on their departure time (i.e. actions with a gap of 5 min, resulting in 72 possible departure times, written in the notation of Section 2 as $A = \{0, 5, 10, ..., 360\}$) taking into account several criteria. The three criteria that will be used at a first stage are

- The time spent on the road.
- The penalty to arrive too late on job.
- The penalty to wake up too early in the morning.

The time spent on the road by the different commuters is deterministically computed following the formula used in Small's study (op. cit.):

$$T_{\text{road}}(t) = T_{\min} + T_{\text{cong}}\left[\frac{A(t)}{R}\right]^{\gamma} \tag{3}$$

where $T_{\text{road}}(t)$ is the time spent on the road to reach the city when leaving home at time t. $T_{\min}$ is the minimum time needed to drive on the road. T_{cong} is a congestion time. $A(t)$ is the total number of commuters at each time t. R is the road capacity, and γ is an elasticity factor. In Mirabel's study (1997), we have $\gamma = 1.4$.

Later on additional criteria can be added in the simulation such as the use of a toll. The use of flexible working hours will be influenced by acting on the parameters of the criteria 'penalty to arrive late on job'.

In what follows we have chosen to evaluate actions directly in terms of preference degrees. The preference functions used for the three aforementioned criteria are respectively shown in Figures 2–4.

The indifference and strict preference parameters of the previous preference functions depend on the profile of the modelled group of commuters. Acting on these parameters will allow us to model different social behaviors. To give the

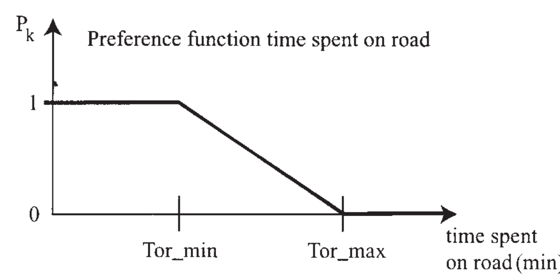

Figure 2. Preference function for the criterion 'Time spent on road'.

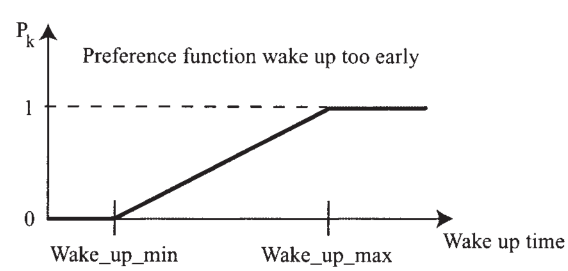

Figure 3. Preference function for the criterion 'Wake up too early'.

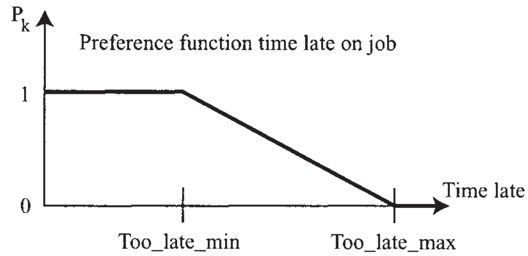

Figure 4. Preference function for the criterion 'Time too late on job'.

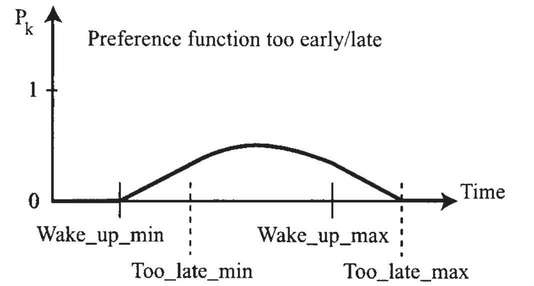

Figure 5. Preference function for the combined criterion 'Time too early/late on job'.

reader an idea of their value, some parameters are listed below for a 'classical' commuter:

- Tor_min = 15', Tor_max = 75'.
- Wake_up_min = 60', Wake_up_max = 210'.
- Too_late_min = 5', Too_late_max = 60'.

Let us note that the values of the 'wake up' parameters are taken with respect to an initial time corresponding to 6 a.m. while those of the 'too late' criterion are taken with respect to the expected arrival time corresponding to 9 a.m.

Furthermore it must be remarked that the criteria 'Wake up too early' and 'Time too late on job' are strongly dependent. Therefore we combine both criteria into a single multiplicative one, with a preference function as shown in Figure 5 with the same parameters as for the uncombined preference functions.

In the next simulations, the relative weights of criteria *time spent on the road and Time too early/late on job* are typically around 10/90%.

It should be noted that the parameters used in the preference model were based on personal feelings and experiences of some of the authors (themselves being commuters), from common sense or from enquiries with colleagues or friends. We still want to emphasize, however, that the objective of presenting this example, is to illustrate the theoretical framework and not to perform a deep analysis of the well-known congestion problem.

3.3. Structure of the simulation model

One of the major drawbacks of our first dynamical multicriteria approach is that an unique decision model is used to represent the behaviors of all the commuters. By doing so, the commuters are supposed to use the same parameters during their decision process. This keeps us from representing, for instance, different, social groups. In order to avoid this, we changed the structure of our simulation setup considerably, by using different agents. Each agent has its own decision parameters and is representing several commuters. Depending on the number of agents a higher degree of heterogeneity can be introduced into the model, which might be interesting to study the responses of different types of car commuters on different policy measures.

The preference matrix is computed as follows:

$$\pi_{ij} = \sum_{k=1}^{n} \omega_k \ \max\{P_k(i) - P_k(j), 0\} \tag{4}$$

where ω_k stands for the weight of the kth criterion and $P_k(i)$ is for the preference degree of alternative a_i on criterion k.

This matrix is then transformed into a probability distribution for the departure times according to Equation (2). Knowing the number of car commuters an agent is representing, the distribution of departures is calculated. This procedure is performed for each agent, which is the first step in our simulation. As a second step, when the departure times are all known, they are communicated to the environment ("the road") in which the agents are positioned. With this information the road "calculates" by means of formula (3) the arrival times of the different car commuters, mixing all the car commuters of the different agents, which is the third step in our simulation procedure. The final step 4 consists in communicating the arrival times back to the agents, so that they can use this information for the next iteration (day) and eventually adapt their strategy. The Figure 6 here below illustrates these four phases. Convergence is expected towards a steady state distribution after a sufficient numbers of days.

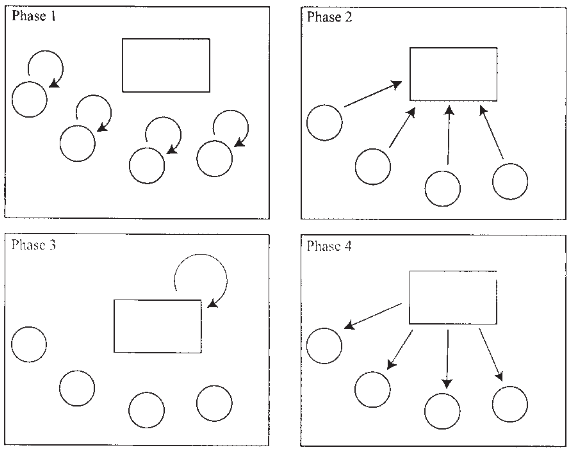

Figure 6. Structure of the simulation step. Phase 1—every agent computes its departure times. Phase 2—agents send their commuters on the road according to their departure decisions. Phase 3—at the end of the day the total number of commuters at each moment of the day and the total time spent on the road is computed. Phase 4—the total time spent on the road is communicated to the different agents. This information will complete the historical experience of each agent and will therefore permit them to improve their departure strategy for the next step.

To conclude, the general structure of the simulation model is a discrete event simulation where each new step (composed by four phases) represents a new day. The result of each step (after phase 4) is used as input for the next step. Agents are using their historical experience to determine the commuters' departure times. Let us remark that when the simulation starts, no historical information is available to the agents. Therefore, we used as initial distribution for 'time spent on road' a randomly generated triangular distribution, centred on 8 a.m. Experiments have shown that this initial assumption has no important impact on the final decision frequencies.

3.4. Results

In this section, we present some results we have obtained with the model described in the previous sections. The aim of these simulations was to obtain some empirical validation of our model. Most of simulations have converged between 80 and 150 days depending on the considered scenario.

3.4.1. Stability of the proposed model. A first simulation was run in order to test the stability of our approach. As already mentioned, our model should represent the decisions of a large number of decision makers (in this case 10^5 commuters). By doing so, we compute a unique preference matrix and exploit it in a stochastic way. This preference matrix is, of course, defined by means of parameters: preference thresholds, criteria weights,... A legitimate question is to wonder whether these parameters have an important impact on the results or not. More precisely, are the decision frequencies obtained through this unique preference matrix (reflecting the homogeneity of the group), similar to those obtained by heterogeneous agents, of which the preference matrix is generated through randomisation of the parameters?

Figure 7 shows the density of departure times obtained by means of 1 (macro), 50, 250, 500 and 1000 heterogenenous agents. As shown the results appear to be rather stable. For each agent, parameters have been randomly perturbed using a uniform distribution. Table I summarizes their parameters.

3.4.2. Four policies. To compare our results with those of previous studies (Small, 1992; Springael et al., 2002), we have applied the four following scenarios: no policy, time flexibility, toll and finally the combination of a toll and time flexibility. Time flexibility has been induced by increasing the 'Too_late_min' and 'Too_late_max' parameters: (15', 120') instead of (5', 60'). In this model, we have considered a toll as a fixed payment in a certain time period. This criterion has been modelled by means of a binary preference function with a toll period from 8 a.m. till 9 a.m. The weight associated to the toll criteria is on average 30% of the total weight. The relative weights between the criteria 'time spent on the road' and 'time too early/too late' are held. The results of the simulations are presented in Figure 8 for a single homogeneous agent.

As expected, both the time flexibility and the toll have an impact on the departure distribution. The effect of the time flexibility policy is a right shift and a widening of the departure times. Applying a fixed toll on a specific period will lead to a congestion reduction for this period but also congestion problems before or after the corresponding time interval. The resulting bi-modal distribution is typical for such policy. In case of

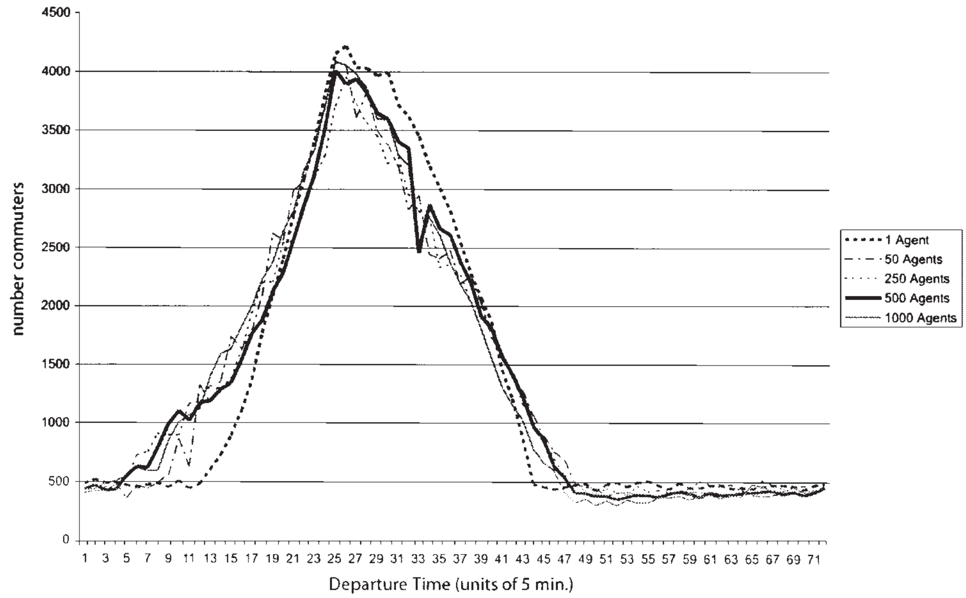

Figure 7. Simulations based on heterogeneous agents. There are 72 possible decisions on the departure time (every 5' starting at 6 a.m. till 12 a.m.).

Table I. Parameters used in the simulations for heterogeneous agents

Parameter	Target value	Min	Max
Weight tor	0.1	0	0.2
Weight too late/early	0.9	0.8	1
Tor_min	15'	fixed	fixed
Tor_max	75'	30'	120'
Wake_up_min	60'	0'	120'
Wake_up_max	210'	150'	270'
Too_late_min	5'	0	10'
Too_late_max	60'	30'	90'

keeping the previous assumptions for the combination of the two policies, it has to be remarked that the congestion after 9 a.m. is much more important than the one before 8 a.m.

3.4.3. Labourers versus office workers. It is well known that every social group does not react in the same manner to time flexibility. Due to the nature of their job, the labourers are less sensitive to the time flexibility than the office workers. The Figure 9 shows the impact of time flexibility on the congestion by considering its influence only on office workers. The labourers/office workers repartition has been assumed to be 30/70%. Labourers are supposed to arrive at work around 8 a.m. while office workers arrive later, i.e. around 9 a.m., and due to their job nature they are supposed to be less flexible: the values Wake_up_min = 0', Wake_up_max = 120', Too_late_min = 0' and Too_late_max = 30' are to be compared with the assumptions given in 3.2.

Considering two heterogeneous social groups leads to macroscopic results that are slightly different from those observed previously. On the other hand, applying time flexibility induces a bimodal curve resulting from the fact that labourers are less sensitive to this policy.

4. CONCLUSIONS AND FUTURE RESEARCH

In this paper, new concepts have been introduced for multicriteria modelling. As mentioned before, we focus here on decision problems where the number of decision makers involved is particularly large. Therefore, the idea of statistical multicriteria decision modelling (SMDM) has been introduced and illustrated through the road traffic congestion problem.

The presented approach is based on the assumption that multicriteria decisions taken by

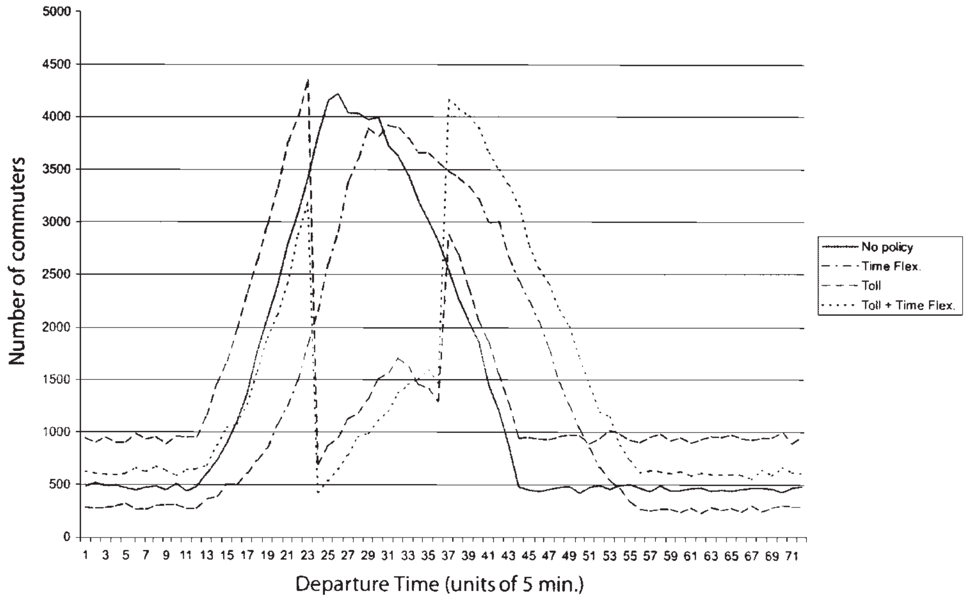

Figure 8. Application of four scenarios (no policy, time flexibility measures, application of a toll and combination of a toll and time flexibility measures) for a single homogeneous agent.

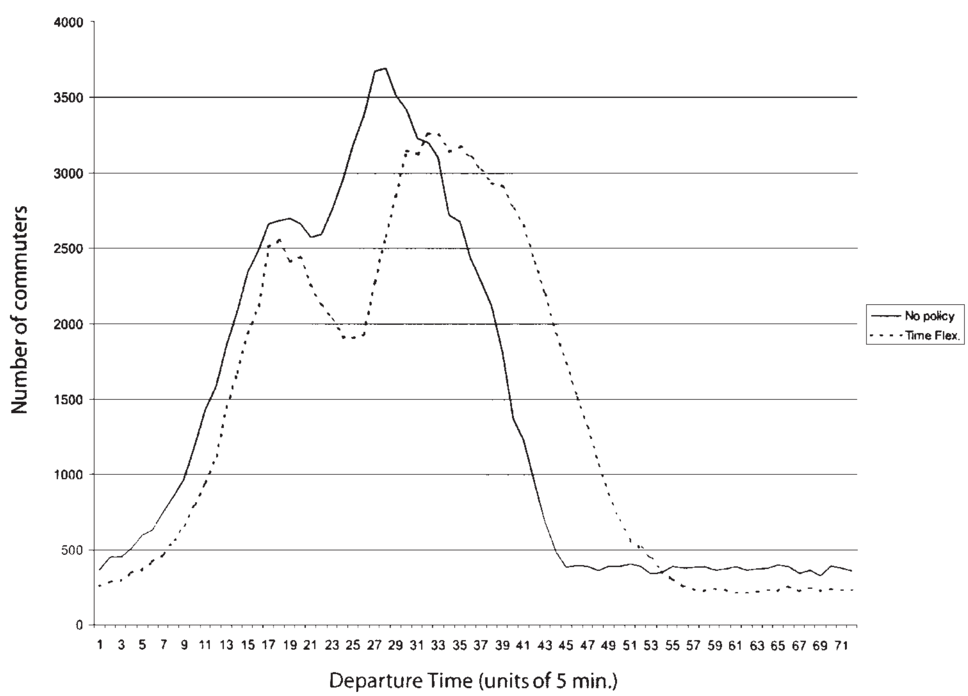

Figure 9. Labourers versus office workers. The curves aggregate the departure times for both labourers and office workers for two scenarios: with or without time flexibility.

a large group of decision makers can be modelled by means of a sufficiently general preference matrix. To model each individual decision, this theoretical preference matrix is exploited by means of Markov chains. Let us note that this unique preference matrix can be computed according to different formalisms, the only restriction imposed on it is that $\pi_{ij} + \pi_{ji} \leqslant 1$ and $\pi_{ij} \geqslant 0, \forall i, j \in A$ Otherwise our model is not linked to any particular multicriteria methodology.

If first results seems encouraging, different questions remain open. Among them, the transformation of preferences into probabilities is certainly the most crucial one. A related question is about the stationary property of the induced Markov chain. Finally, applying this approach to other real life situations will permit to further test its coherence.

From the multicriteria point of view, some questions raised in Springael et al. (2002) still deserve some attention:

- Should the MCDA-agent perform a full pairwise comparison of all actions, or, on the contrary, a partial comparison of subsets of possible actions?
- How to represent the preference information within the 'homogeneous' group of decision makers?

With respect to this last question we have tried to give an answer in the way the simulation was conceived. For each agent, representing an homogeneous group of decision makers, different preference degrees and weights were stochastically selected. Nevertheless the simulation results may lead to the conclusion that this effect on the final distribution is a minor one.

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