



University of Antwerp  
Operations Research Group

ANT/OR

# The Collaborative Selective Vehicle Routing Problem

## Vehicle routing in a collaborative environment

Christof Defryn    Kenneth Sörensen

University of Antwerp – ANT/OR Operations Research Group

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# Overview

The Selective Vehicle Routing Problem

Introduction to the Collaborative Environment

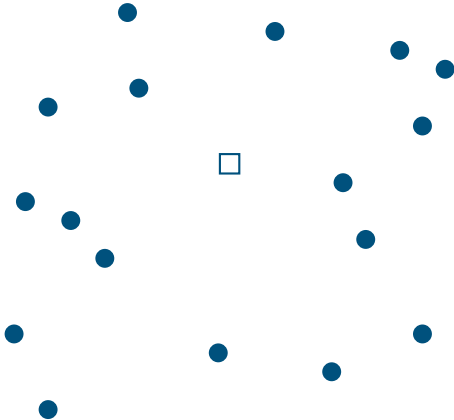
Solving of the Collaborative Problem

- Strategic positioning

- Gain Sharing (or Cost Allocation)

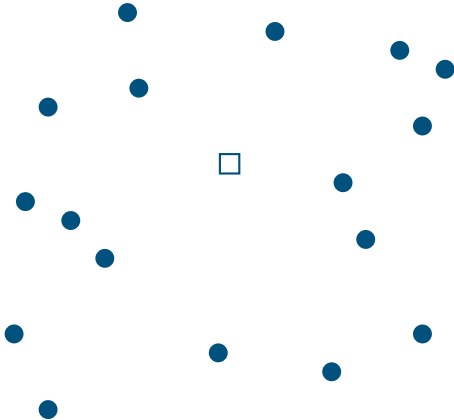
A simulation study

Concluding Remarks



### Imagine...

- ▶ A central depot
- ▶ A set of  $N$  clients, waiting to be served

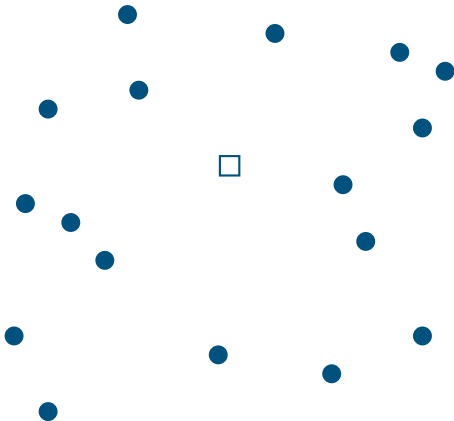


### Imagine...

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### But...

- ▶ Only a limited number of resources (*trucks*) is available
- ▶ Usage of a truck is limited (*distance, time*)



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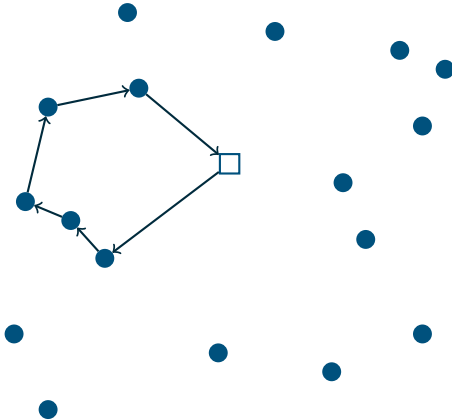
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### Therefore...

- ▶ Only a limited number of clients can be served now



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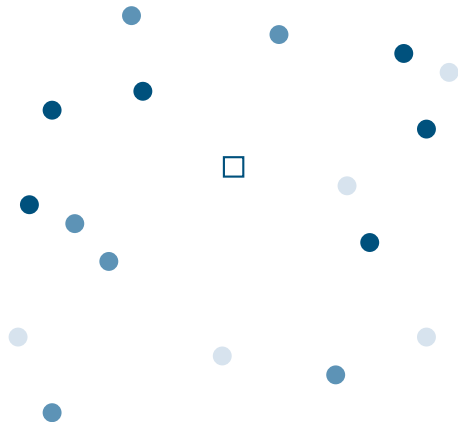
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## The Collaborative Environment introduced

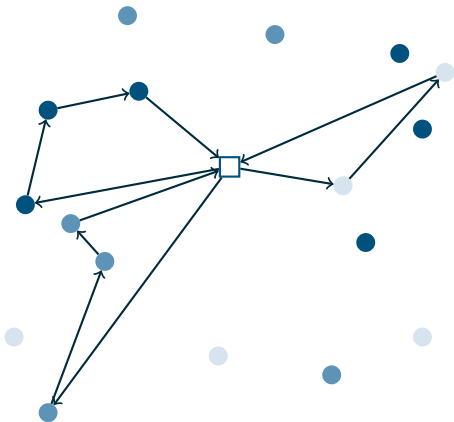


### Imagine...

- ▶ A central depot
- ▶ 3 partners, each having a set of clients to be served
- ▶ Each partner possesses one vehicle



## The Collaborative Environment introduced



### Global Result

- ▶ 8 clients are served (2-3-3)

### Introduce Collaboration

- ▶ The three partners form a strategic alliance
- ▶ One vehicle can serve clients from different partners in the same trip





## The Collaborative Environment introduced



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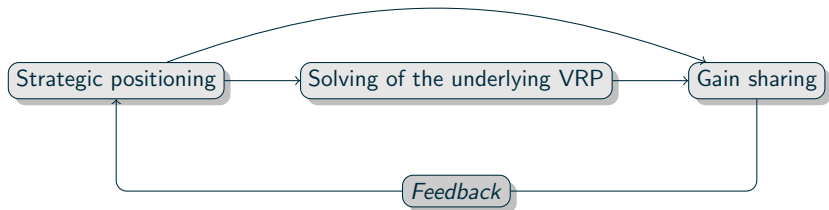
### Global Result Collaboration

- ▶ 11 clients are served (3-3-5)



## A collaboration is more than calculating routes

- ▶ Companies remain independent entities
  - ▶ Results will be evaluated only on **personal gains**
  - ▶ Every partner wants his (important) clients to be part of the solution





## How do I **behave** in the coalition?

*“Whatever the result will be, I agree”*

versus

*“All my (important) clients should be part of the solution”*

The partners are given the possibility to set a **cost** for all of their clients, that is to be paid by the group if the client is **not taken into the final solution**



## How do I **behave** in the coalition?

*“Whatever the result will be, I agree”*

versus

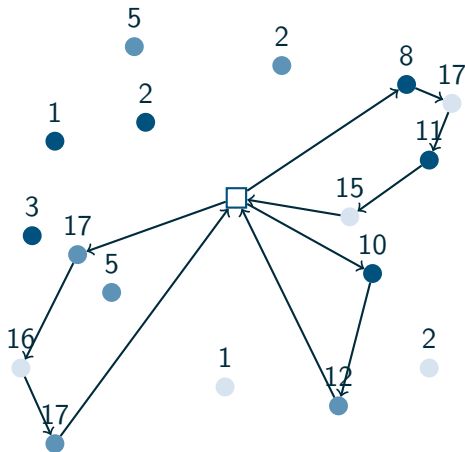
*“All my (important) clients should be part of the solution”*

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**Compensation for non-delivery (CND)**



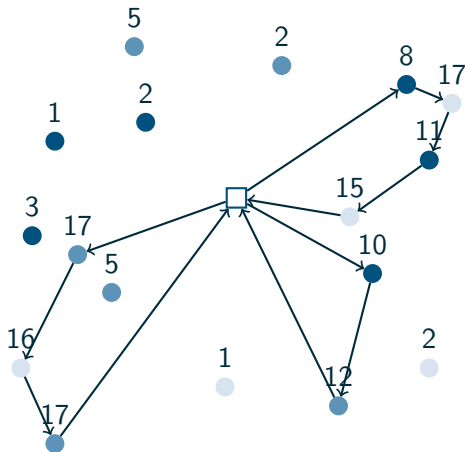
## How do I behave in the coalition?



- ▶ Different CND for every client
- ▶ Clients with a higher cost are more likely to be part of the solution
- ▶ Significant differences in CND will pull the solution away from maximal efficiency in favour of the expensive clients
- ▶ Number of clients visited will lower



## How do I **behave** in the coalition?



### Global Result

- ▶ 8 clients are served (2-3-3)

### Global Result Collaboration

- ▶ 11 clients are served (3-3-5)

### Global Result Collaboration (CND)

- ▶ 9 clients are served (3-3-3)



## Divide and Conquer

*“Setting up a coalition should be profitable”  
“How much of the coalition cost should I pay?”  
“Coalition gains should be divided in a **fair** way”*

The cost allocation method should give an incentive to the partners to behave **flexible**.

- ▶ CND is cost for coalition and should be kept as low as possible



## Divide and Conquer

- ▶ We compare two different cost allocation approaches
  - ▶ Shapley Value
  - ▶ CND-weighted allocation





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### Shapley Value

$$x_i = \sum_{S \subseteq N \setminus \{i\}} \frac{|S|!(n - |S| - 1)!}{n!} (v(S \cup \{i\}) - v(S))$$

- ▶ Put forward by the CO<sup>3</sup>-consortium as possible **best practice** (standard)
- ▶ Cost allocation based on the **impact** of a partner on every subcoalition
- ▶ Ability to reward **flexible behaviour**



## Divide and Conquer

- ▶ We compare two different cost allocation approaches
  - ▶ Shapley Value
  - ▶ CND-weighted allocation

### CND-weighted allocation

$$x_i = M_i + \frac{CND_i}{\sum_i CND_i} (C(N) - \sum M_i)$$

- ▶ Separable cost ( $M_i$ ) + weighted division of remaining cost
- ▶ Only based on **specific** colSVRP parameters (*client location & CND policy*)



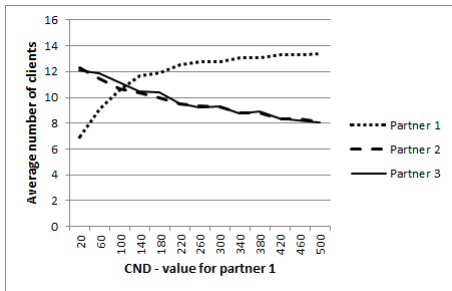
## A simulation study

- ▶ Own-generated Test instances
- ▶ 3-partner coalitions
- ▶ Different scenarios
  - ▶ Impact of variable CND
  - ▶ Distance from the depot
  - ▶ Client Clustering
- ▶ Partner 1 has a variable compensation for non-delivery (CND) cost, for partners 2 and 3 the CND is fixed to 100
- ▶ For every variable setting, results are averaged over 30 different instances



## Impact of Compensation for non-delivery

*Client locations are generated randomly*

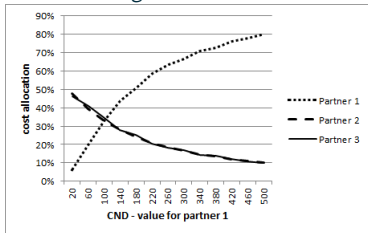


- ▶ If CND-values are the same, an equal number of clients is served
- ▶ Raising the CND makes your clients more important, more clients will be taken into the final solution

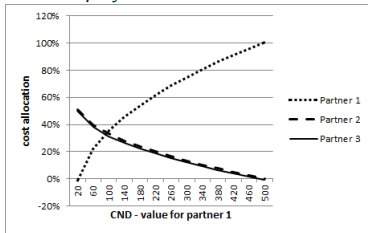


# Impact of Compensation for non-delivery

*CND-weighted cost allocation*



*Shapley Value cost allocation*

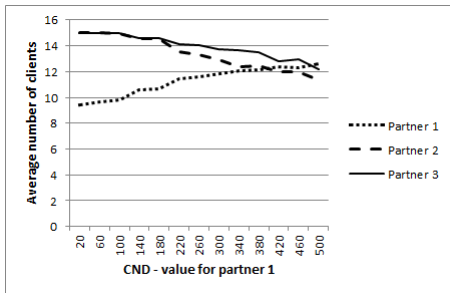


- ▶ Similar behaviour of both allocation methods
- ▶ Shapley punishes inflexibility more



## Distance from depot

*Clients of Partner 1 are located relatively far from the depot*

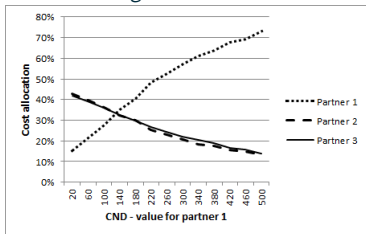


- ▶ If CND-values are the same, partners with more accessible clients will be favoured
- ▶ Unfavoured clients need to be significant expensive (high CND value) to make the longer trip valuable

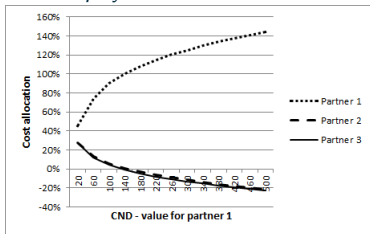


## Distance from depot

*CND-weighted cost allocation*



*Shapley Value cost allocation*

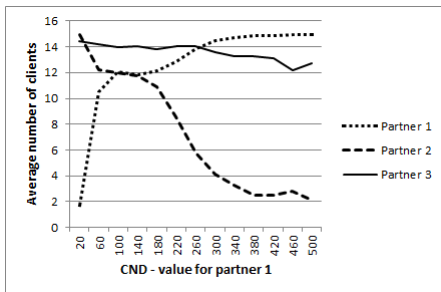


- ▶ Forcing the clients of partner 1 in the solution increases the coalition cost, which is penalized in both methods
- ▶ Due to non-monotonic increasing costs, allocations above 100% and below 0% can be found with Shapley



## Client Clustering

Clients belonging to the same partner are clustered in one region of the solution space. On average, clients of Partner 3 are located closer to the depot.



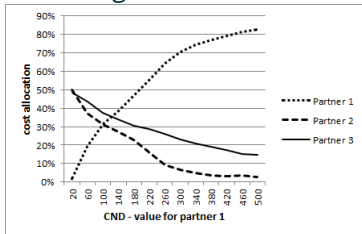
- ▶ The cluster closest to the depot is favoured (*see Scenario 2*)
- ▶ If clients are located further, they need to be significant expensive to be favoured
- ▶ Favouring one partner is at the expense of the furthest, flexible partner



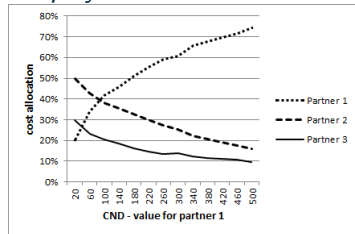


# Client Clustering

*CND-weighted cost allocation*



*Shapley Value cost allocation*



- ▶ CND-weighted method is based on the final routing solution. Less clients in the solution → lower cost (Partner 2)
- ▶ Shapley Value is based on all sub-coalitions. Closer to the depot → higher stand-alone efficiency → lower cost (Partner 3)



## To conclude

- ▶ Joining forces enables to exploit new synergies
- ▶ A collaborative problem is more than the sum of the partners
- ▶ The behaviour of partners affects both **optimal route and cost allocation**
- ▶ The interaction between cost allocation and routing should not be neglected



## To conclude

Shapley Value CA	CND-weighted CA
✓ An established method recognized by the industry	✓ Direct incentive towards flexibility for the colSVRP
✓ Efficiency of all sub-coalitions	✓ Easy to understand/calculate
× Equal service in final solution may not be equally charged to the partners	✓ Cost is never negative or above 100% of coalition cost
× Allocations above 100% and below 0% possible	× Only based on the final solution
	× Parameters other than client location and CND are neglected



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