The Clustered Vehicle Routing Problem

a two-level variable neighbourhood search

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Outline

Problem description

Metaheuristic approach
  Construction
  Intensification
  Diversification

Results
About the Clustered VRP
About the Clustered VRP

Assume...

- A central depot
- A set of N clients, waiting to be served
- A set of V homogeneous vehicles with capacity Q
About the Clustered VRP

Assume...
- A central depot
- A set of N clients, waiting to be served
- A set of V homogeneous vehicles with capacity Q

But...
- Clients are clustered
- Clients belonging to the same cluster should be visited by the same vehicle sequentially in the same path
Metaheuristic approach
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Metaheuristic approach
1. **Construction (Cluster level)**
   - Randomized Bin Packing Problem

2. **Intensification**
   - VNS at cluster level
   - Conversion operator
   - VNS at customer level

3. **Diversification**
   - Perturbation
   - Mutation
   - Restart
Metaheuristic approach

1. **Construction (Cluster level)**
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Construction - Cluster level

- Given:
  - Number of vehicles
  - Capacity of each vehicle
  - Demand of a cluster

- Allocate all clusters to a vehicle

- One-dimensional Bin Packing Problem

- Heuristic methods:
  - First-fit decreasing heuristic
  - Best-fit decreasing heuristic
Construction - Cluster level

- Given:
  - Number of vehicles
  - Capacity of each vehicle
  - Demand of a cluster

- Allocate all clusters to a vehicle

- One-dimensional Bin Packing Problem

- Heuristic methods:
  - First-fit decreasing heuristic
  - Best-fit decreasing heuristic
    - Best = Close to clusters already in the vehicle
    - Consider the nbBest best actions (randomness)
1. **Construction (Cluster level)**
   - Randomized Bin Packing Problem

2. **Intensification**
   - VNS at cluster level
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3. **Diversification**
   - Perturbation
   - Mutation
   - Restart
Intensification - Cluster level (VNS - 1)

- Four **intra vehicle** local search operators
  - Swap (*swap position of two clusters*)
  - Relocate (*remove one cluster, insert it at a different position*)
  - Two-Opt (*remove two edges, close the tour with two new edges*)
  - Or-Opt (*remove N sequential clusters, insert them at a different position, with $N = \{2, 3, 4\}$*)

- Three **inter vehicle** local search operators
  - Swap (*swap two clusters from different vehicles*)
  - Relocate (*remove a cluster from a vehicle and insert it in another vehicle*)
  - Or-Opt (*remove N sequential clusters, insert them in another vehicle, with $N = \{2, 3, 4\}$*)
Intensification - Convert to individual clients

- 0 → D → C → B → 0
- Based on the Sweep Heuristic
- Intra-cluster client order
  - Clients are sorted according to their arctan - value

Based on the Sweep Heuristic
Intra-cluster client order
Clients are sorted according to their arctan - value
Intensification - Convert to individual clients

- Based on the Sweep Heuristic
- Intra-cluster client order
  - Clients are sorted according to their arctan-value
- Inter-cluster strategy
  - Go to the closest client
  - Only consider the first and the last client of a cluster

\[0 \rightarrow D \rightarrow C \rightarrow B \rightarrow 0\]
Intensification - Convert to individual clients

- 0 → D → C → B → 0
- Based on the Sweep Heuristic
- Intra-cluster client order
  - Clients are sorted according to their arctan - value
- Inter-cluster strategy
  - Go to the closest client
  - Only consider the first and the last client of a cluster
Intensification - Client level (VNS - 2)

- Four **intra cluster** local search operators
  - Swap *(swap position of two clients within a cluster)*
  - Relocate *(remove one client and insert it at a different position within the same cluster)*
  - Two-Opt *(remove two edges and close the tour with two new edges)*
  - Or-Opt *(remove N sequential clients, insert them at a different position within the same cluster, with N = \{2, 3, 4\})*

- Two local search operators at **cluster level**, questioning the optimal cluster sequence *(inter + intra vehicle)*
  - Swap *(swap position of two clusters)*
  - Relocate *(Remove all clients of a given cluster and insert them at a different position)*
Intensification - Client level (VNS - 2)

- Clusters should be kept together!
- If we are not able to improve the solution with the given neighbourhoods → Local Optimum
- If the solution is better than the best solution found until now, it becomes the new best solution
Metaheuristic approach

1. **Construction (Cluster level)**
   - Randomized Bin Packing Problem

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   - Perturbation
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Diversification

▶ **Perturbation**
  ▶ Randomly delete 50% of the clusters in the current solution
  ▶ A new solution is then constructed by adding clusters to a random (possible) vehicle

▶ **Mutation**
  ▶ Randomly make a small change (*swap*) to the solution at cluster level
  ▶ Mutated solution is immediately improved at individual client level

▶ **Restart**
  ▶ If no improvement is found after 100 iterations
  ▶ Clear the full solution and restart the complete algorithm again
Results

- **Small and Medium sized instances**
  - 79 GVRP instances test instances - *Bektas et al. (2011)*
  - Denoted as GVRP$\theta_3$
  - Solved as CluVRP to optimality - *Battarra et al. (2014)*

- **Large instances**
  - 20 sets of instances - *Battarra et al. (2014), based on Golden(1998)*
  - Each set consists of 11 instances with a variable number of clusters
  - Optimal results of the instances are known and will be used as a benchmark
Small and Medium sized instances

Results for the GVRPθ3 instances. Comparison between the branch and cut and price (BCP), branch and cut (BC) (*battarra et al. 2014*) and the proposed two-level variable neighbourhood approach (VNS).

<table>
<thead>
<tr>
<th></th>
<th>BCP</th>
<th></th>
<th>BC</th>
<th></th>
<th>VNS</th>
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<td></td>
<td>Opt.</td>
<td>CPU (s)</td>
<td>Opt.</td>
<td>CPU (s)</td>
<td>Opt.</td>
<td>CPU (s)</td>
</tr>
<tr>
<td>A (27 inst)</td>
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<td>42.52</td>
<td>27</td>
<td>4.84</td>
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<td>0.16</td>
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<tr>
<td>B (23 inst)</td>
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<td>23</td>
<td>4.99</td>
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<td>24</td>
<td>3.77</td>
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<tr>
<td>M+G (5 inst)</td>
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<td>4</td>
<td>25.44</td>
<td>0</td>
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<td>78</td>
<td>5.86</td>
<td>48</td>
<td>0.37</td>
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average
Large instances

Results for the large-sized Golden instances. Comparison between the branch and cut algorithm with graph reduction and initial solution as upper bound (BC(GR+UB)), and the variable neighbourhood search (VNS)

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<tr>
<th>θ</th>
<th>BC (GR+UB)</th>
<th>VNS</th>
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<td>15 (20 inst)</td>
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<table>
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To conclude

- An efficient algorithm is developed, for solving the Clustered Vehicle Routing Problem.
- By exploiting the clustered substructure of the problem and by implementing the two-level approach, the total complexity of the problem is reduced.
- In very short calculation times, good quality solutions are obtained for instances up to 484 nodes.
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