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ANT/OR

The Clustered Vehicle Routing Problem a two-level variable neighbourhood search

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Outline

Problem description

Metaheuristic approach

- Construction

- Intensification

- Diversification

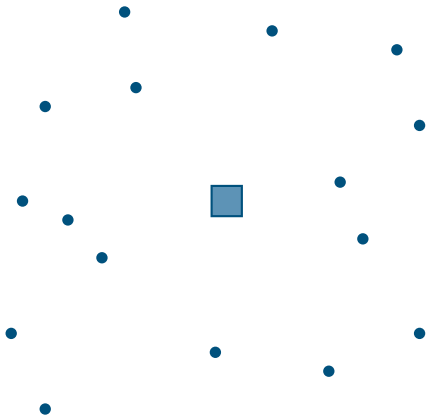
Results



About the Clustered VRP



About the Clustered VRP

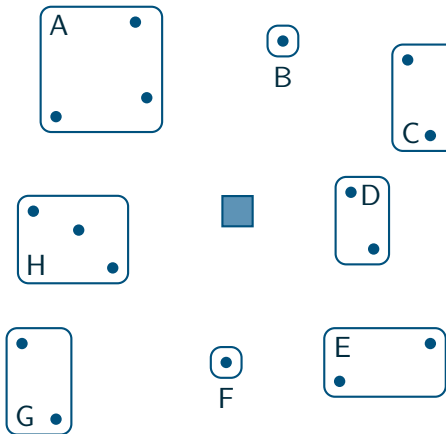


Assume...

- ▶ A central depot
- ▶ A set of N clients, waiting to be served
- ▶ A set of V homogeneous vehicles with capacity Q



About the Clustered VRP



Assume...

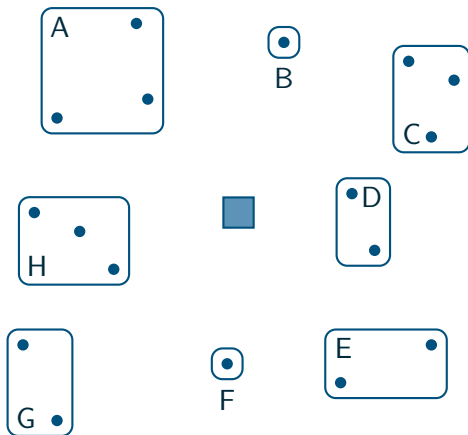
- ▶ A central depot
- ▶ A set of N clients, waiting to be served
- ▶ A set of V homogeneous vehicles with capacity Q

But...

- ▶ Clients are clustered
- ▶ Clients belonging to the same cluster should be visited by the same vehicle sequentially in the same path

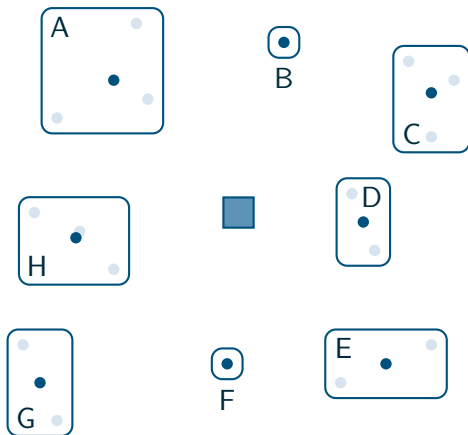


Metaheuristic approach



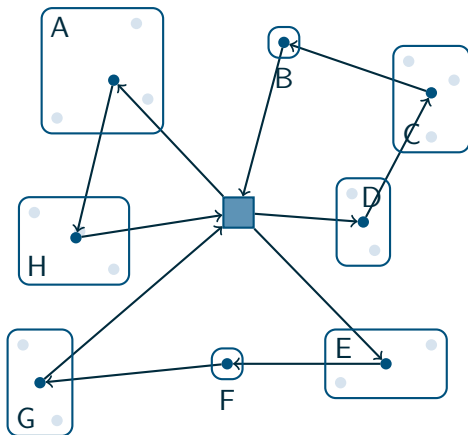


Metaheuristic approach



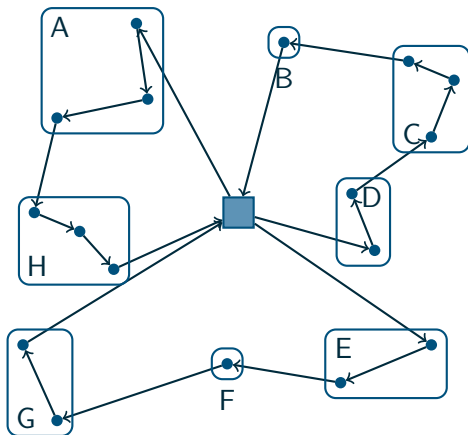


Metaheuristic approach





Metaheuristic approach



1. Construction (Cluster level)

- ▶ Randomized Bin Packing Problem

2. Intensification

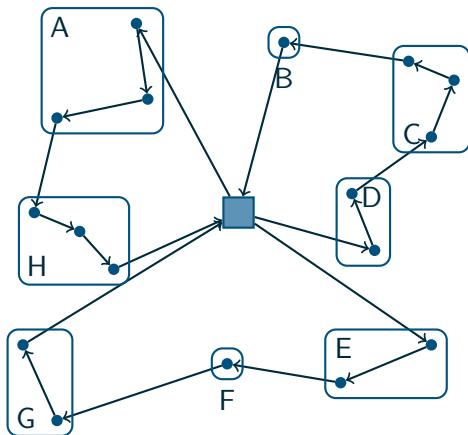
- ▶ VNS at cluster level
- ▶ Conversion operator
- ▶ VNS at customer level

3. Diversification

- ▶ Perturbation
- ▶ Mutation
- ▶ Restart



Metaheuristic approach



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Construction - Cluster level

- ▶ Given:
 - ▶ Number of vehicles
 - ▶ Capacity of each vehicle
 - ▶ Demand of a cluster
- ▶ Allocate all clusters to a vehicle
- ▶ **One-dimensional Bin Packing Problem**
- ▶ Heuristic methods:
 - ▶ First-fit decreasing heuristic
 - ▶ Best-fit decreasing heuristic

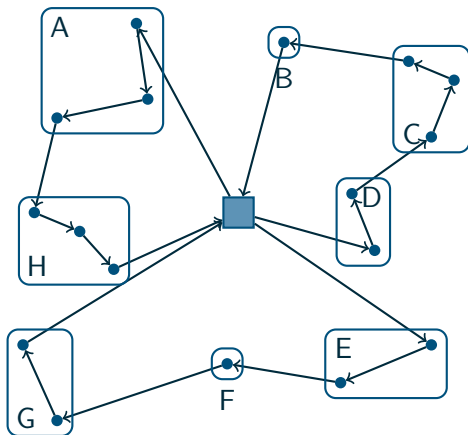


Construction - Cluster level

- ▶ Given:
 - ▶ Number of vehicles
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- ▶ Allocate all clusters to a vehicle
- ▶ **One-dimensional Bin Packing Problem**
- ▶ Heuristic methods:
 - ▶ First-fit decreasing heuristic
 - ▶ Best-fit decreasing heuristic
 - ▶ *Best = Close to clusters already in the vehicle*
 - ▶ *Consider the nbBest best actions (randomness)*



Metaheuristic approach



1. **Construction (Cluster level)**
 - ▶ Randomized Bin Packing Problem
2. **Intensification**
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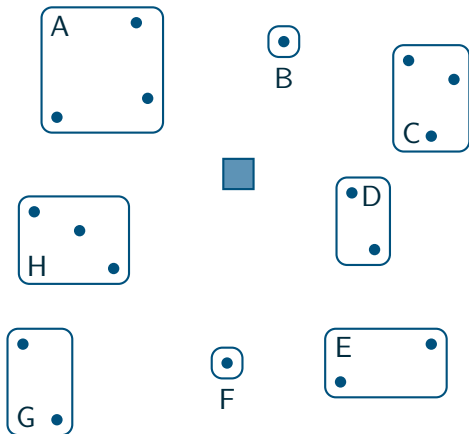


Intensification - Cluster level (VNS - 1)

- ▶ Four **intra vehicle** local search operators
 - ▶ Swap (*swap position of two clusters*)
 - ▶ Relocate (*remove one cluster, insert it at a different position*)
 - ▶ Two-Opt (*remove two edges, close the tour with two new edges*)
 - ▶ Or-Opt (*remove N sequential clusters, insert them at a different position, with $N = \{2, 3, 4\}$*)
- ▶ Three **inter vehicle** local search operators
 - ▶ Swap (*swap two clusters from different vehicles*)
 - ▶ Relocate (*remove a cluster from a vehicle and insert it in another vehicle*)
 - ▶ Or-Opt (*remove N sequential clusters, insert them in another vehicle, with $N = \{2, 3, 4\}$*)



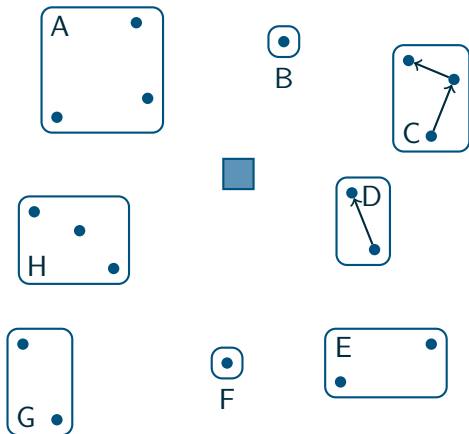
Intensification - Convert to individual clients



- ▶ $0 \rightarrow D \rightarrow C \rightarrow B \rightarrow 0$
- ▶ Based on the **Sweep Heuristic**
- ▶ Intra-cluster client order
 - ▶ Clients are sorted according to their arctan - value



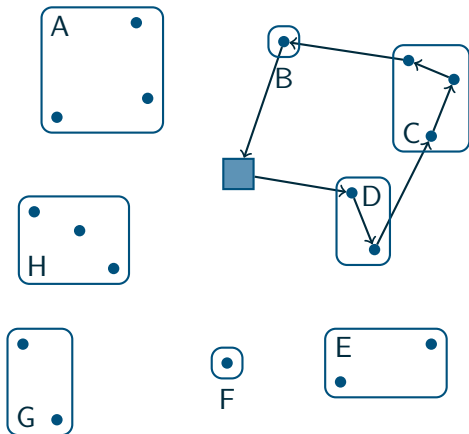
Intensification - Convert to individual clients



- ▶ $0 \rightarrow D \rightarrow C \rightarrow B \rightarrow 0$
- ▶ Based on the **Sweep Heuristic**
- ▶ Intra-cluster client order
 - ▶ Clients are sorted according to their arctan - value
- ▶ Inter-cluster strategy
 - ▶ Go to the closest client
 - ▶ Only consider the first and the last client of a cluster



Intensification - Convert to individual clients



- ▶ $0 \rightarrow D \rightarrow C \rightarrow B \rightarrow 0$
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- ▶ Intra-cluster client order
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- ▶ Inter-cluster strategy
 - ▶ Go to the closest client
 - ▶ Only consider the first and the last client of a cluster



Intensification - Client level (VNS - 2)

- ▶ Four **intra cluster** local search operators
 - ▶ Swap (*swap position of two client within a cluster*)
 - ▶ Relocate (*remove one client and insert it at a different position within the same cluster*)
 - ▶ Two-Opt (*remove two edges and close the tour with two new edges*)
 - ▶ Or-Opt (*remove N sequential clients, insert them at a different position within the same cluster, with $N = \{2, 3, 4\}$*)
- ▶ Two local search operators at **cluster level**, questioning the optimal cluster sequence (*inter + intra vehicle*)
 - ▶ Swap (*swap position of two clusters*)
 - ▶ Relocate (*Remove all clients of a given cluster and insert them at a different position*)

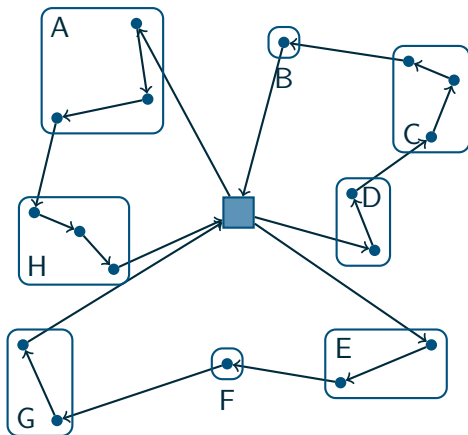


Intensification - Client level (VNS - 2)

- ▶ Clusters should be kept together!
- ▶ If we are not able to improve the solution with the given neighbourhoods → **Local Optimum**
- ▶ If the solution is better than the best solution found until now, it becomes the new best solution



Metaheuristic approach



1. **Construction (Cluster level)**
 - ▶ Randomized Bin Packing Problem
2. **Intensification**
 - ▶ VNS at cluster level
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3. **Diversification**
 - ▶ Perturbation
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Diversification

▶ Perturbation

- ▶ Randomly delete 50% of the **clusters** in the current solution
- ▶ A new solution is then constructed by adding clusters to a random (possible) vehicle

▶ Mutation

- ▶ Randomly make a small change (*swap*) to the solution at cluster level
- ▶ Mutated solution is immediately improved at individual client level

▶ Restart

- ▶ If no improvement is found after 100 iterations
- ▶ Clear the full solution and restart the complete algorithm again



Results

▶ **Small and Medium sized instances**

- ▶ 79 GVRP instances test instances - *Bektas et al. (2011)*
- ▶ Denoted as GVRP θ 3
- ▶ Solved as CluVRP to optimality - *Battarra et al. (2014)*

▶ **Large instances**

- ▶ 20 sets of instances - *Battarra et al. (2014)*, based on *Golden(1998)*
- ▶ Each set consists of 11 instances with a variable number of clusters
- ▶ Optimal results of the instances are known and will be used as a benchmark



Small and Medium sized instances

Results for the GVRP θ 3 instances. Comparison between the branch and cut and price (BCP), branch and cut (BC) (*battarra et al. 2014*) and the proposed two-level variable neighbourhood approach (VNS).

		BCP		BC		VNS		
		Opt.	CPU (s)	Opt.	CPU (s)	Opt.	CPU (s)	GAP
A (27 inst)	31 - 79 cust.	27	42.52	27	4.84	19	0.16	0.12%
B (23 inst)	30 - 77 cust.	23	7.69	23	4.99	17	0.21	0.11%
P (24 inst)	15 - 100 cust.	24	0.48	24	3.77	12	0.21	0.24%
M+G (5 inst)	100 - 261 cust.	2	157.25	4	25.44	0	2.92	1.46%
total (79 inst)		76		78		48		
average			26.87		5.86		0.37	0.22%



Large instances

Results for the large-sized Golden instances. Comparison between the branch and cut algorithm with graph reduction and initial solution as upper bound (BC(GR+UB)), and the variable neighbourhood search (VNS)

θ	BC (GR+UB)		VNS		
	Opt.	CPU (s)	Opt.	CPU (s)	GAP
5 (20 inst)	17	363.00	0	4.60	3.96%
6 (20 inst)	19	86.65	0	3.36	3.49%
7 (20 inst)	19	109.71	0	2.54	3.01%
8 (20 inst)	19	93.86	0	2.23	2.93%
9 (20 inst)	19	92.18	0	2.01	2.88%
10 (20 inst)	20	48.82	0	1.86	2.78%
11 (20 inst)	20	72.23	0	1.77	2.99%
12 (20 inst)	20	48.71	0	1.71	2.83%
13 (20 inst)	20	40.20	0	1.81	2.71%
14 (20 inst)	20	39.18	0	1.80	2.76%
15 (20 inst)	20	23.09	0	1.83	2.68%
total average	213	88.66	0	2.32	3.00%



To conclude

- ▶ An efficient algorithm is developed, for solving the Clustered Vehicle Routing Problem.
- ▶ By exploiting the clustered substructure of the problem and by implementing the two-level approach, the total complexity of the problem is reduced.
- ▶ In very short calculation times, good quality solutions are obtained for instances up to 484 nodes



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a two-level variable neighbourhood search

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