University of Antwerp
Operations Research Group
ANT/OR

The Clustered Vehicle Routing Problem

a two-level variable neighbourhood search

Christof Defryn Kenneth Sörensen
University of Antwerp – ANT/OR Operations Research Group

EURO/IFORS 13-18th July 2014 – Barcelona

Research supported by the Research Foundation - Flanders (FWO - Ph.D. fellowship) and the Interuniversity Attraction Poles (IAP) Programme initiated by the Belgian Science Policy Office (COMEX project)

Outline

Problem description

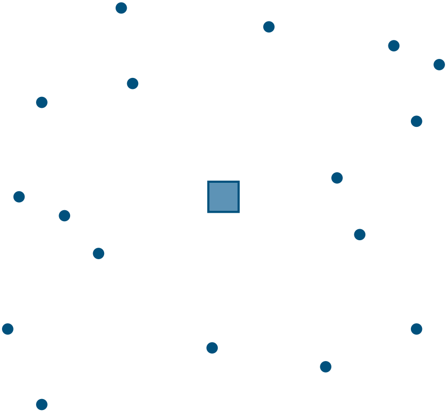
Metaheuristic approach

- Construction
- Intensification
- Diversification

Results

About the Clustered VRP

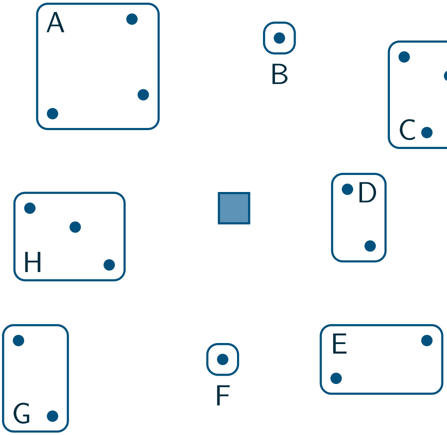
About the Clustered VRP



Assume...

- A central depot
- A set of N clients, waiting to be served
- A set of V homogeneous vehicles with capacity Q

About the Clustered VRP



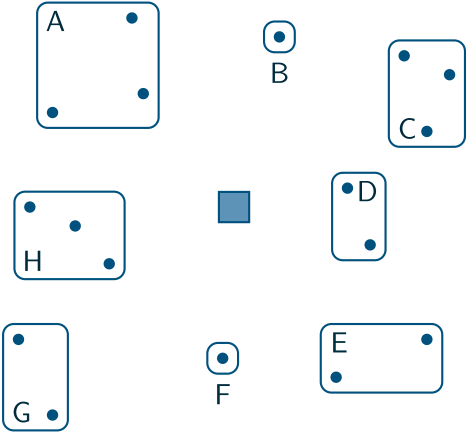
Assume...

- A central depot
- A set of N clients, waiting to be served
- A set of V homogeneous vehicles with capacity Q

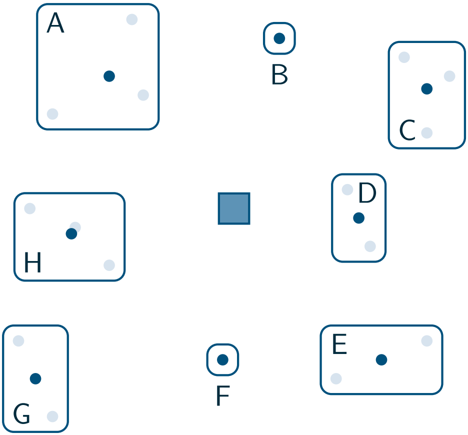
But...

- Clients are clustered
- Clients belonging to the same cluster should be visited by the same vehicle sequentially in the same path

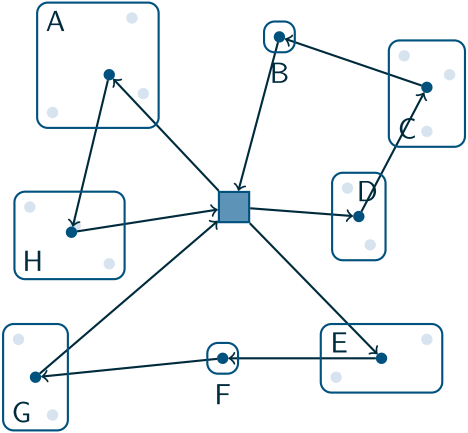
Metaheuristic approach



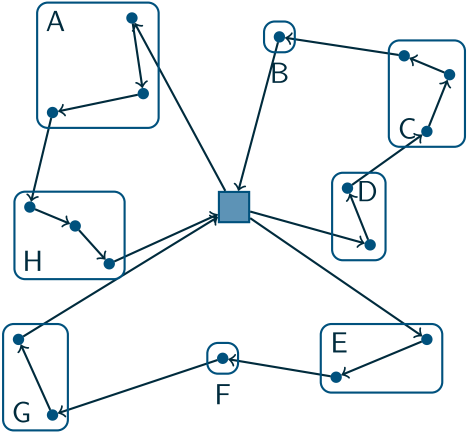
Metaheuristic approach



Metaheuristic approach

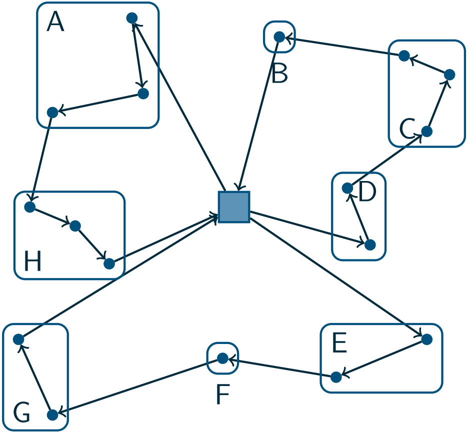


Metaheuristic approach



1. **Construction (Cluster level)**
   - Randomized Bin Packing Problem
2. **Intensification**
   - VNS at cluster level
   - Conversion operator
   - VNS at customer level
3. **Diversification**
   - Perturbation
   - Mutation
   - Restart

Metaheuristic approach



1. **Construction (Cluster level)**
   - Randomized Bin Packing Problem
2. **Intensification**
   - VNS at cluster level
   - Conversion operator
   - VNS at customer level
3. **Diversification**
   - Perturbation
   - Mutation
   - Restart

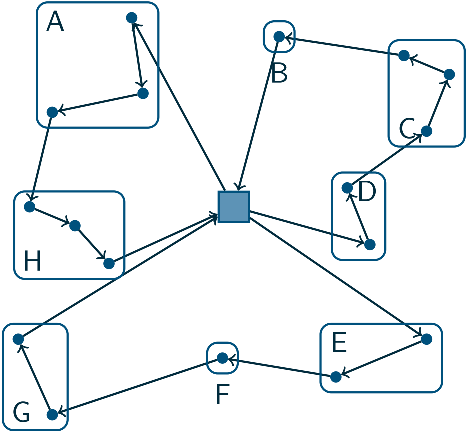
Construction - Cluster level

- Given:
  - Number of vehicles
  - Capacity of each vehicle
  - Demand of a cluster
- Allocate all clusters to a vehicle
- **One-dimensional Bin Packing Problem**
- Heuristic methods:
  - First-fit decreasing heuristic
  - Best-fit decreasing heuristic

Construction - Cluster level

- Given:
  - Number of vehicles
  - Capacity of each vehicle
  - Demand of a cluster
- Allocate all clusters to a vehicle
- **One-dimensional Bin Packing Problem**
- Heuristic methods:
  - First-fit decreasing heuristic
  - Best-fit decreasing heuristic
    - *Best = Close to clusters already in the vehicle*
    - *Consider the nbBest best actions (randomness)*

Metaheuristic approach

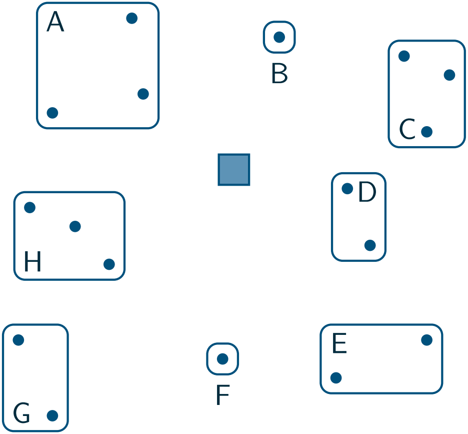


1. **Construction (Cluster level)**
   - Randomized Bin Packing Problem
2. **Intensification**
   - VNS at cluster level
   - Conversion operator
   - VNS at customer level
3. **Diversification**
   - Perturbation
   - Mutation
   - Restart

Intensification - Cluster level (VNS - 1)

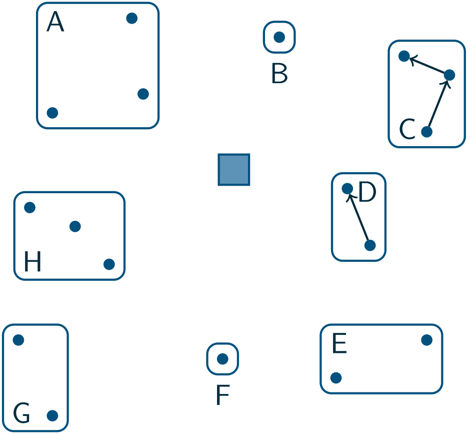
- Four **intra vehicle** local search operators
  - Swap *(swap position of two clusters)*
  - Relocate *(remove one cluster, insert it at a different position)*
  - Two-Opt *(remove two edges, close the tour with two new edges)*
  - Or-Opt *(remove N sequential clusters, insert them at a different position, with $N = \{2, 3, 4\}$)*
- Three **inter vehicle** local search operators
  - Swap *(swap two clusters from different vehicles)*
  - Relocate *(remove a cluster from a vehicle and insert it in another vehicle)*
  - Or-Opt *(remove N sequential clusters, insert them in another vehicle, with $N = \{2, 3, 4\}$)*

Intensification - Convert to individual clients



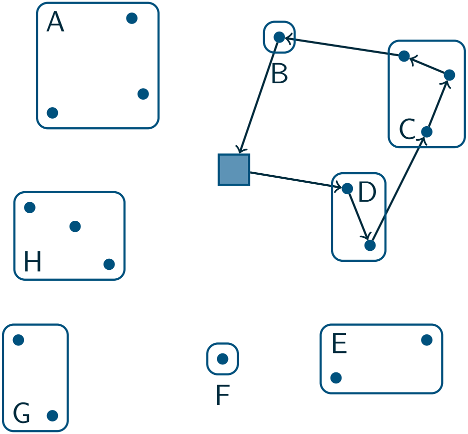
- **0 → D → C → B → 0**
- Based on the Sweep Heuristic
- Intra-cluster client order
  - Clients are sorted according to their arctan - value

Intensification - Convert to individual clients



- $\mathbf{0} \rightarrow \mathbf{D} \rightarrow \mathbf{C} \rightarrow \mathbf{B} \rightarrow \mathbf{0}$
- Based on the Sweep Heuristic
- Intra-cluster client order
  - Clients are sorted according to their arctan - value
- Inter-cluster strategy
  - Go to the closest client
  - Only consider the first and the last client of a cluster

Intensification - Convert to individual clients



- **0 → D → C → B → 0**
- Based on the Sweep Heuristic
- Intra-cluster client order
  - Clients are sorted according to their arctan - value
- Inter-cluster strategy
  - Go to the closest client
  - Only consider the first and the last client of a cluster

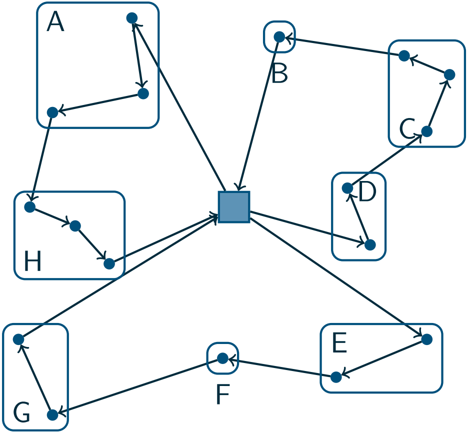
Intensification - Client level (VNS - 2)

- Four **intra cluster** local search operators
  - Swap *(swap position of two client within a cluster)*
  - Relocate *(remove one client and insert it at a different position within the same cluster)*
  - Two-Opt *(remove two edges and close the tour with two new edges)*
  - Or-Opt *(remove N sequential clients, insert them at a different position within the same cluster, with $N = \{2, 3, 4\}$)*
- Two local search operators at **cluster level**, questioning the optimal cluster sequence *(inter + intra vehicle)*
  - Swap *(swap position of two clusters)*
  - Relocate *(Remove all clients of a given cluster and insert them at a different position)*

Intensification - Client level (VNS - 2)

- Clusters should be kept together!
- If we are not able to improve the solution with the given neighbourhoods $\rightarrow$ Local Optimum
- If the solution is better than the best solution found until now, it becomes the new best solution

Metaheuristic approach



1. **Construction (Cluster level)**
   - Randomized Bin Packing Problem
2. **Intensification**
   - VNS at cluster level
   - Conversion operator
   - VNS at customer level
3. **Diversification**
   - Perturbation
   - Mutation
   - Restart

Diversification

- **Perturbation**
  - Randomly delete 50% of the clusters in the current solution
  - A new solution is then constructed by adding clusters to a random (possible) vehicle
- **Mutation**
  - Randomly make a small change (*swap*) to the solution at cluster level
  - Mutated solution is immediately improved at individual client level
- **Restart**
  - If no improvement is found after 100 iterations
  - Clear the full solution and restart the complete algorithm again

Results

- **Small and Medium sized instances**
  - 79 GVRP instances test instances - *Bektas et al. (2011)*
  - Denoted as GVRPθ3
  - Solved as CluVRP to optimality - *Battarra et al. (2014)*
- **Large instances**
  - 20 sets of instances - *Battarra et al. (2014), based on Golden(1998)*
  - Each set consists of 11 instances with a variable number of clusters
  - Optimal results of the instances are known and will be used as a benchmark

Small and Medium sized instances

Results for the GVRPθ3 instances. Comparison between the branch and cut and price (BCP), branch and cut (BC) *(battarra et al. 2014)* and the proposed two-level variable neighbourhood approach (VNS).

		BCP		BC		VNS		
		Opt.	CPU (s)	Opt.	CPU (s)	Opt.	CPU (s)	GAP
A (27 inst)	31 - 79 cust.	27	42.52	27	4.84	19	0.16	**0.12%**
B (23 inst)	30 - 77 cust.	23	7.69	23	4.99	17	0.21	**0.11%**
P (24 inst)	15 - 100 cust.	24	0.48	24	3.77	12	0.21	**0.24%**
M+G (5 inst)	100 - 261 cust.	2	157.25	4	25.44	0	2.92	**1.46%**
total (79 inst)		76		78		48		
average			26.87		5.86		0.37	0.22%

Large instances

Results for the large-sized Golden instances. Comparison between the branch and cut algorithm with graph reduction and initial solution as upper bound (BC(GR+UB)), and the variable neighbourhood search (VNS)

	BC (GR+UB)		VNS		
θ	Opt.	CPU (s)	Opt.	CPU (s)	GAP
5 (20 inst)	17	363.00	0	4.60	**3.96%**
6 (20 inst)	19	86.65	0	3.36	**3.49%**
7 (20 inst)	19	109.71	0	2.54	**3.01%**
8 (20 inst)	19	93.86	0	2.23	**2.93%**
9 (20 inst)	19	92.18	0	2.01	**2.88%**
10 (20 inst)	20	48.82	0	1.86	**2.78%**
11 (20 inst)	20	72.23	0	1.77	**2.99%**
12 (20 inst)	20	48.71	0	1.71	**2.83%**
13 (20 inst)	20	40.20	0	1.81	**2.71%**
14 (20 inst)	20	39.18	0	1.80	**2.76%**
15 (20 inst)	20	23.09	0	1.83	**2.68%**
total	213		0		
average		88.66		2.32	**3.00%**

To conclude

- An efficient algorithm is developed, for solving the Clustered Vehicle Routing Problem.
- By exploiting the clustered substructure of the problem and by implementing the two-level approach, the total complexity of the problem is reduced.
- In very short calculation times, good quality solutions are obtained for instances up to 484 nodes

University of Antwerp
Operations Research Group
ANT/OR

The Clustered Vehicle Routing Problem

a two-level variable neighbourhood search

Christof Defryn Kenneth Sörensen
University of Antwerp – ANT/OR Operations Research Group
EURO/IFORS 13-18th July 2014 – Barcelona

Research supported by the Research Foundation - Flanders (FWO - Ph.D. fellowship) and the Interuniversity Attraction Poles (IAP) Programme initiated by the Belgian Science Policy Office (COMEX project)

christof.defryn@uantwerpen.be